Final Reflection

By Dolly

**all names and identifiers have been masked/changed to retain anonymity*

Introduction

The fraction orange began as an exploration into the mathematical thinking surrounding fractions. Its design was inspired by the struggles surrounding fractions I had witnessed in the classroom among multiple students, more specifically with my focus student, Jay. As his thinking revealed itself in our interviews, I began to see that his understanding of fractions was confused and disconnected through his drawings and explanations. From my perspective, fractions seemed like a scary mystery to Jay; they were something to throw every recondite math fact and procedure he knew at, in hope that one might help him arrive at the “right” answer. Jay was suffering from what many other children in our school system have suffered -- he began to “unlearn” his intuitive understanding of fractions in exchange for models that represent fractions in restricted ways. He needed something that could forge connections and offer a tangible, formative experience with fractions so that he could make his own meaningful connections across his own mathematical thinking and experiences (Empson & Levi, 2011). Jay needed something that would allow him to arrive at answers from his own wonderings and discovery, rather than by the thoughts, procedures, and products of someone else’s thinking. He needed a fraction orange.

The purpose of the fraction orange started simply as a tool to help children understand how parts can interact with their whole in order to help them conceptualize dealing with fractions on a deeper level. However, once it came to be as a physical manipulative, it started to reveal itself as being much more elegant and clever than I had originally expected. The experience of working with the fraction orange found its own kind of relevance in the same way art often does: “that of being a meaningful human experience” (Lockhart, 2009, p. 9). It came to elicit from its user a thoughtful examination of relationships, mathematical and otherwise; but most importantly it became a meaning-making tool with the purpose of facilitating an understanding of the beautiful, complex, relational nature of fractions through concrete experience, and ultimately to lead to flexibility in abstract thought and mathematical thinking.

Design and Idea Rationale

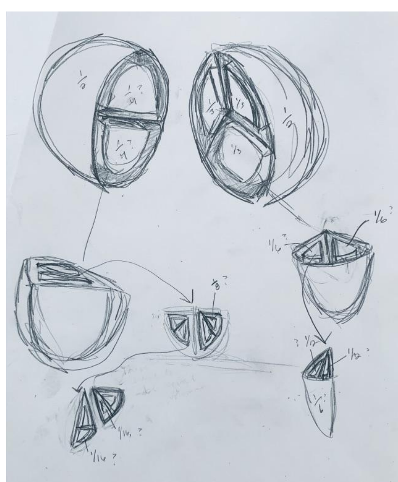
The design of the orange came from an exploration of my own experiences. I thought about what kinds of objects are generally pleasant to hold, and I settled on a sphere. The slicing of the sphere naturally lead to the concept of an orange. From there, I thought about the design from the perspective of the future user. As they start to uncover (peel away) the different compartments of the orange, they will discover how a whole can be divided into different equal parts (slices). As they halve the whole orange, they will discover two different sides. Through exploring the different halves, the user will begin to realize that one half is divided equally into two parts, and the other is divided equally into three parts. I will call them them the “half of twos” and “half of threes” for the purpose of this paper (see image)

As the user explores the half of twos, they will be able take the next smallest pieces (now dealing with quarters of the whole orange, or halves of the half), and remove them from their “nests”. Upon doing so, they will discover that those quarters are also divided into halves (which become eighths of the whole, or quarters of the half), and so on until they discover the pieces that are sixteenths of the whole, or eighths of the half. They will be able to do the same with the half of threes, discovering that the pieces come apart first as thirds of the half (or sixths of the whole), and then sixths of the half (or twelfths of the whole). The user will be able to completely deconstruct and reconstruct the orange, lay its fractions or parts side-by-side, or see how the parts and slices may or may not fit into other parts, etc.

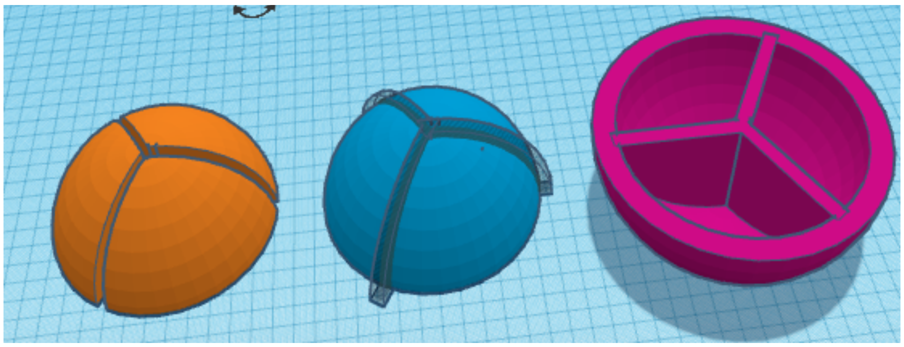
Through physically dealing with the different slices of the half of twos and the half of threes, the user will hopefully begin to see the relationship between parts and how they can come together to make a whole. The nesting is an important part of the design because it puts forth a sense of discovery for the user through the "unnesting" process, and hopefully gives them an opportunity to have an "ah-ha!" moment regarding how a whole can be decomposed into parts, or fractions, and how they might fit back together in an exciting way. The frame, or "skeleton" of the whole or larger part that remains after its parts are removed also serves as a reminder to the user that the parts they are now holding came from the decomposition of the whole before them. The orange will also hopefully show the user what division of fractions can look and feel like. For example, they will be able to see that half of the half of twos will produce the same two pieces, which are simultaneously quarters of the whole and halves of the half. Therefore, they can make the connection that a whole divided by four is the same as a half divided into halves. The orange begins to show the user the relational nature of fractions; that wholes that are divided into fractions of two (or three) can actually become the parts or wholes of one another. Through exploration and interaction with the physically realized design, the user will hopefully grow in their thinking about fractions in a consequential and (literally and figuratively) global way.

It was important for me to break half of the orange into halves and the other half in thirds, as well as to find a way to make them all nest within one another three-dimensionally. I feel that the comparison of the halves and thirds and how they ultimately fit into the same whole gives the user a basis for flexible thinking in terms of how objects can translate into different fractions within itself, and how written fractions can translate into a whole composed of different parts, even within itself. One of my original hesitations about the design was the challenge that arose from having two different halves, as it asks the user to conceptualize a double of each in order to make sense of the pieces in relation to the *whole sphere*, rather than just the half. However, I hoped that gap in information could be useful to the user because it might push them to *imagine* the other half. If not, however, I decided on a double printing of the orange for children (or adults), who many need the concrete experience of the whole orange of halves or thirds in order to make sense of their thinking. I also would have liked to have included one more division of the half of threes, but ultimately it was not realistic to design and I feel that it is currently effective enough to demonstrate the previously mentioned learning points. In addition, the smaller pieces would be too cumbersome to physically manipulate and would not add enough value to make the object bigger or redesign it in order to accommodate them. After printing and holding the orange in my own hands, and along with with watching others physically interact with it, I realized a few design flaws (but hopefully nothing that takes away from its usability). The first flaw is the awkward nature of putting the two halves together to make a whole sphere, and the inevitable chaos of re-opening them. In a redesign, I would create a cover that could contain all of the slices of the orange in each half, without it interfering with the experience of closing the orange. Second is the difficulty of getting the pieces (particularly the smaller ones) out of their "nests". This proved to be a difficult task for adult men, however children could navigate the pieces with relative ease.

The actual design process in Tinkercad was intuitive and improvisational. I originally began the design process with a sketch (see image below),



as I normally do, to get the first draft out of my mind and onto paper. From there, I began with the largest pieces first, which also happened to be the easiest to create with the shapes provided in Tinkercad. I went into constructing each piece of the orange with an idea of how I might arrive at its design, and from there I continued to tinker to find the best solutions. Ultimately, I thought of each piece as I do when creating sculptural pieces by hand -- as a process of addition and subtraction, building and reduction. I made many duplicates of each piece in their beginning phases so that I could compare iterations and try different techniques without having to undo and redo steps. By using the hole tool, I was able to use shapes as 3D erasers that could "carve" away the shapes I needed. The hole tool also made it possible for me to see my shapes inside one another to ensure that they would nest properly once printed. To create my frames, or skeletons, I sliced parts of existing duplicate pieces to use as dividing walls. I hardly used the measurement tool, and most of the design choices were made by visual judgement. The half of twos was easily accomplished by using the alignment tool to find the halfway point of many of the shapes. Some of my biggest design challenges came from creating the half of threes, when I needed to figure out how to divide up half of a sphere as equally as possible into thirds. I used my understanding that a circle is 360 degrees, divided it by 3 to reach 120 degrees, and took the dividing walls I had made from slicing half spheres and intersected them with one another at 120 degree angles. I then used the hole tool and rectangular prisms to remove half of each, and then was able to use that corresponding shape as a hole to create my third slices, and as a solid to create my skeleton (see below).



There were many other design choices that I navigated through as they arrived, but the entire process was very intuitive and I never felt that I encountered a challenge I could not solve by thinking about it deeply enough.

Key Design Decisions

My design choices were informed by my own experiences. As stated before, when I began to think about this project, I knew I wanted to think about fractions in a naturally interactive context. After thinking of which situations or things I have commonly divided into parts throughout my life, I arrived at the orange. I knew I wanted the design to be an interactive sphere, not a glorified representation of the two dimensional fraction circle that haunt the minds of many children and adults. The answer was, of course, to make an orange (in shape, slices, and color). I also gravitated towards creating a spherical object because I believe people enjoy interacting with spheres (i.e. sports, ball pits, bubbles, globes, etc.). So, the design of the fraction orange began. I wanted to make an object that was familiar on a conscious and subconscious level, because those are the types of experiences that people borrow from when they are making connections in learning.

My original design had the user “discovering” the pieces inside of each section by fully removing it from its bigger holder. The pieces inside originally would not be visible unless the user took the orange apart. However, after discussing the concept with an individual who conceptualized my design as creating a half sphere with a flat surface that could be divided into parts (more like a glorified fraction circle -- the pieces would be flat and rest on the surface of the half-sphere), I did realize the value in being able to see all the parts in one view. I decided to readjust the design so that all of the pieces were accessible at once when the user had the orange in their hand. With all of that in mind, I felt that my fraction orange would be an exciting and pleasant manipulative to use, and would hopefully encourage students to use it to explore their own thinking and understandings of fractions.

One of the most important aspect of the design that I struggled with was dividing the orange into "equal" parts. I feared that the typical relationship people have with fractions, one of performing algorithms without understanding their meaning, might cause some misunderstandings with the orange and its parts (i.e. 9 of the ninth slices, when combined, do not match the volume of the whole). I was afraid that this rigid thinking of what a fraction was might interfere with the meaning of the manipulative, and therefore its effectiveness. In addition, the slices, or fractions, are not perfectly equal to one another because I chose to divide them up visually rather than by exact measurements. I made this decision because dividing into parts is messy sometimes, especially when we do it in a real context (like slicing an orange or breaking apart food). In those real-life contexts, the pieces are not *exactly equal* to one another, but they are still understood as such; this makes the orange more relatable as an object and the process more practical, and therefore the concept more accessible. Related to this idea is the choice to make the slices "nest" in one another, which then meant a need for a framework or skeleton to hold the pieces in. This framework is one of the reasons that the slices were not equal or proportionally sized in relationship to the whole (by making them exist, the pieces inside of them had to be even smaller). This choice was one that can be an advantage or disadvantage in that it asks the user to understand the parts, or fractions, in a more representative or symbolic way rather than a literal way. Hopefully, this interplay between symbolism and reality might help the user bridge those connections as well when they are using it to understand abstract ideas in practical contexts.

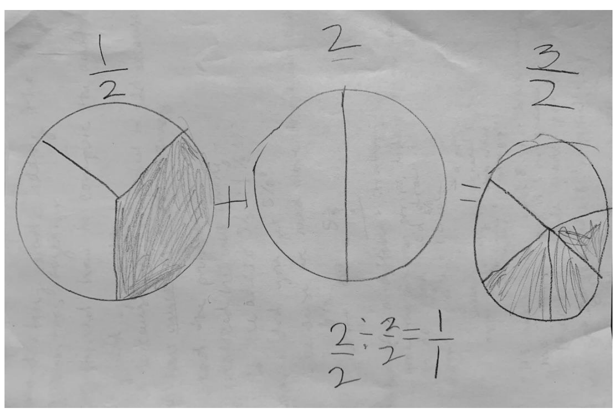
Task Statement

I designed the following two problems for Jay:

1. Chance ate $\frac{4}{8}$ of his pizza. Janelle ate $\frac{1}{2}$ of her pizza. Who ate more?
2. Justin has a birthday cake. He wants to divide it equally between his 9 friends. How much cake does each friend get?

I designed each of these tasks because I thought they complimented the nature of the manipulative. My intention for using a comparing fractions problem was drawn from the fact that I knew that Jay had been “doing” them in class, but only through algorithms of finding common denominators. In addition, the fraction orange was beneficial for comparison in that it maintains its framework (skeleton) as a visual reference while its pieces are laid by its side. When I envisioned the student using the orange to solve this problem, I hoped he might use the half of twos to discover the eighths pieces, and then hopefully realize that four of them fill the same space in the orange as a half. Some of the struggles I anticipated centered around Jay’s confusion about the whole in his fractional endeavors, as he often lost sight of which piece his whole actually was when thinking about fractions (which could be a confusing aspect of the orange, too).

The equal sharing birthday cake problem was designed because I knew that my student struggled deeply with fractions, and never had a proper representation of what fractions really look like or mean in his mind, as can be seen by the following example of his work:



I wanted to give him an equal sharing problem, as they are a useful and intuitive type of problem in order to develop fractional understandings (which unfortunately is what Jay needed). This intuitive understanding of sharing, measuring, and distribution would help provide a foundation for knowledge that he could later attach symbols to, rather than trying to make meaning from using symbols that he did not understand in the first place (Empson & Levi, 2011). Using the manipulative in this case would allow him to use the side of threes to find a physical representation of what one ninth would look like, or hopefully to even make the connection that one ninth is one third of a third. I had hoped that by using the fraction orange, he would be able to divide a whole into ninths, and understand what one-ninth actually means in relation to its whole. I hoped to foster mathematical proficiency to build “adaptive expertise”, or the ability to apply meaningful knowledge flexibly and thoughtfully to novel tasks (Baroody, 1987, pg. 17). In other words, by combining his knowledge with the concrete manipulative experience, I hoped that Jay would build a deeper understanding of fractions so he could handle them in a flexible, improvisational way in his mind.

My professor mentioned that I might use the fraction orange with adults. I loved this idea, so I designed 3 tasks to guide adult interaction with it. They were:

1. $1 \div 9$
2. $2 \div \frac{1}{4}$
3. $\frac{1}{2} \div \frac{1}{4}$

I chose the first question because I wanted to present adults with a similar problem I presented to Jay. The second and third problem were designed with the hopes that the adults would solve them using an algorithm (i.e. multiplying by the reciprocal), and would then be able to explain why that process works through the manipulative. I chose to make the tasks equations rather than word problems because I wanted them to be straight-forward, and I assumed that most adults already had a solid basis of understanding of fractions (I was wrong). Therefore, I did not need to use word problems to create meaningful contexts for them to utilize in their problem solving. I wanted to challenge what they knew to find out how deep their understandings were about fractions. I began with a whole number divided by a whole, then a whole divided by a fraction, and finally a fraction divided by a fraction, because it seemed to be a natural progression of “comfort” or cognitive demand. I presented the tasks to them one-by-one.

Findings

I conducted 9 interviews with the fraction orange and one session of free-play and exploration. I will begin with my interview with Jay, as he was the inspiration for the orange from the beginning. The first task was to compare equal fractions. Jay struggled with this the most, which was surprising to me. I introduced the manipulative to Jay and explained that he could make any piece represent the whole. This seemed to bewilder Jay as he worked, as he kept changing what the whole was (at one part of the interview he pointed to the entire orange, during another he pointed to a half). Jay would not write anything down, and sat with the half of twos in his hand. He would begin to take out a piece, and say "wait", and then put it back. This continued for minutes, until I tried to ask non-guiding questions to facilitate some sort of progress. I asked him, what would four-eighths look like with the orange? He sat quietly. He took the quarter pieces out, and finally the smaller pieces from them (the eighths) and placed them in front of him. He said, "these are the eighths". I asked him why he thought that, and he told me "because there are 8 of them in this piece". This implied that Jay could represent what it meant to break up a whole into eighths, or 8 parts. He took 4 of the 8 pieces, and told me "that's how many Chance ate". This implied to me that he understood the connection between the written fraction and the relational values it represented. From there, he went on to try to explain the half. He chose a quarter piece instead of the half, and then seemed bewildered at how he might use that information to answer the question. I asked him again what piece would show him what ½ of a pizza would be. He held up the half piece. I said, "who ate more?" and he froze again. This showed me that he was either struggling to understand what it meant to compare the fractions, or that the orange's symbolic partitions were confusing to him. He began picking up the small 8th pieces and holding them against the large half piece. They weren't equal, obviously, and I believe it caused him to hesitate. After some time, though, he said "Oh yeah! Yeah they did eat the same amount. Because inside [the other half piece] it's one, two, three, four [counting the 8ths that were nested into the quarter pieces of that half], and that's the same as this one [referencing the half that he was holding in his hand]". Finally, Jay was able to make the connection between the relationships that fractions have to each other and to their whole, and the barrier the fraction orange may have set up with its disproportionate sizes was overcome.

The second task was an equal sharing problem with a fraction as an answer. As we remember from his previous sessions, Jay's understanding of fractions were confused and inconsistent, and he relied, almost desperately, on algorithms taught to him by his teacher. When I read the problem to Jay, he almost immediately told me "one ninth" as his answer. However, when I asked him to show me what one ninth would look like with the fraction orange, he stalled. This showed me that he might not understand what one ninth actually means. He took one of the thirds out of the half of threes. He said, "Each friend would get one third." I asked him, "One third of what?" and he held up the larger third piece. He was right, it was a third of a third of the whole. However, I believe he still had a disconnect in recognizing that his answer and "one-ninth" were one in the same, because he kept saying "wait...wait" and shuffling pieces around. However, after sitting with the orange and counting the nine pieces, he decided that it was one out of nine pieces, and the nine pieces made up the whole. I asked him to reiterate which piece was the whole, and he hesitated before answering that the half of threes represented the whole. After he was given time to sit with the pieces and navigate through his thinking, he arrived at a (hopefully) deeper understanding of what these fractions actually represent.

The same day I interviewed Jay with the fraction orange, I also brought it to my kindergarten class. I presented it to the little group of students who constantly asked me if we "could do math together." They enthusiastically took it and began to explore. They immediately started disassembling the orange, and I asked, "who thinks they know what a half is?" and all of the children could point to a half. However, one student took the half in his hand, and discovered that *it also had halves*. Even more excitingly, he discovered that *those halves even had halves*. He exclaimed this discovery with pride, and I told him, overjoyed, that he had figured out how to make quarters and eighths. Other children did the same with the half of threes, as one little girl told me that there were “three pieces in each of the three big pieces, and that there were nine of them all together.” She had discovered thirds and ninths. After I explained to her what she discovered, she told me that she “knew that it broke into pieces but she didn’t know what it was called,” implying that she had an innate sense of fractions, but did not have the language to articulate it yet.

So far, the fraction orange had proven itself as a useful explanatory and exploratory mathematical manipulative. However, its role in becoming a humbling catalyst for self-reflection in adults was by far the most exciting and unexpected part of its unfolding identity. Using it in interviewing adults was astounding. I Interviewed 8 adults from the ages of 26 to 72 (their education levels ranged from an associate degree to doctor of veterinary medicine). When I presented the tasks, in order, one at a time, most of the adults (5 out of 8) were able to reach the correct answer on paper through the algorithms they were taught in school. However, when I asked those adults to explain their reasoning or process with the manipulative, only 3 out of those correct 5 were able to explain it in a way that demonstrated a deeper understanding, or even simply an understanding, of the meaning of fractions or division with fractions. The one-ninth problem was often described as being “too simple”, however almost everyone was able to initially identify a ninth slice as representing their answer. When I asked why the small ninth piece they chose represented their answer, they seemed to get confused (many of them said things like “wait...maybe I’m wrong”, and “it *is* this one, right?”).

In addition, asking adults what dividing a number by a fraction actually meant solicited many reactions, including “Why did I say yes to this?”, and “There is no explanation!”. Ultimately, though, the fraction orange allowed at least a few of the adults to assign meaning to the algorithms they originally used to solve the problem. One of the doctors explained it when he said “So, if you said how many quarters are in a half, the answer is obviously two. But in my head I don’t really think of that as the same thing as a half divided by a quarter...and the orange helped me to see that.”

One of the most common threads throughout the adult interviews was of an attitude of "not being a math person", which accompanied a rigid type of thinking that interfered with their ability to use the manipulative to reason through their answers. Working with adults with this attitude was interesting, because it was like seeing an end-product to the self-fulfilling myth of fixed, innate math abilities. It is my hope that the orange could help adults challenge their fixed mindsets, and help them to adopt a malleable perception of intelligence by giving them tools to expand their understandings (Kimball & Smith, 2013). The relationships held between the pieces of the orange called on adults to come face-to-face with their misunderstandings (or lack of any understanding), of what different equations meant or even what the meaning of a fraction was. In addition, some of the adults also struggled with the fact that the slices represent symbolic fractions of the whole, not the exact proportions of those fractions, and one explained to me that she would have done better if I had explained that she could decide which piece represented a whole for each problem (I gave the problems and orange to the adults with no explanation). It challenged adults into a disequilibrium, and asked them to remake their own understandings of what fractions were and what division with fractions actually meant.

Reflections

This design experience turned out to be an exciting and inspiring endeavor for me. Much like my fraction orange, it uncovered multiple layers of my understanding of not only fractions, but of learning in general, and how one makes connections between mathematical thinking and concrete experiences. While I have designed and created many objects in the past, I had never done so with the technology of 3D printing, nor for the intention of mathematical learning. If I were to experience this again in the future, I might want to know the context of *why* 3D printing was chosen as the medium for the project, and perhaps more discussion about the importance of not only designing a math manipulative, but also the value that comes from creating and using it in a classroom context. I would have preferred to have those discussions prior to the design process in order to lay down that framework with which to experience the remaining rest of the Maker experience.

I discovered that designing something for learning requires a unique focus and perspective in the creative process, and it was a challenge to learn how to balance the utility and practicality of the object while still infusing wonder and exploration into its use. That being said, I am pleased with how my design turned out. I am happy with my design decisions to keep the orange's parts divided in a "human", albeit not exactly equal, way. There were also parts of the design that I had not considered that manifested themselves when it became a tangible object, that turned out to be quite delightful. For example, the kinetics of the orange, how the pieces move within it, combined with the sounds that come from using it added another enjoyable dimension of sensory experience to its use.

I also learned through this design experience coupled with my experience as a student teacher that both processes require an incredible amount of thinking and planning, but also tinkering. A design cannot be successful if all the parts of its whole were not thought out...their relationships and interactions must be considered which requires a deep understanding of the material, the making process, and the execution of the design. This is also true of teaching; a lesson will not be successful without a deep knowledge of the material being taught, the teaching process, or how to implement it in the classroom. The creative process of trial and error that runs through all of it, and reflection on that process, is vital to creating new ideas and understandings that can be transferred into the classroom. Van de Walle, Karp, and Bay-Williams (2013) describe the process well:

> "To construct or build something in the physical world requires tools, materials, and effort. The tools we use to build understanding are our existing ideas and knowledge. The materials we use to build understanding may be things we see, hear, or touch, or our own thoughts and ideas. The effort required to connect new knowledge to old knowledge is reflective thought." (pg. 20)

It is through this tinkering and reflective thought that we can have productive and honest conversations with ourselves, which will in turn inform a better design, a more captivating lesson, or deeper understandings.

These productive and honest conversations also were evident in the interviews, both between myself and the subject, but also between individuals and themselves. While it might sound obvious, as an interviewer I learned how important it is to ask questions, and also how important it is to give space to and truly listen to the person speaking. It is within that space that you can actually begin to understand their thinking, which is an incredibly useful insight to have to inform future teaching. That being said, I know that I probably guided the interview too much because I was personally invested into it, and I really wanted it to be helpful to my student. In the future, I would respect the inquiry and exploration that accompanies mathematical thinking, and give it more space to do so.

The interview process was valuable to the student, I believe, because he started to forge new understandings of fractions that he did not have before. The exploratory play with the fraction orange was also valuable for the kindergarteners, as it exposed them to the fact that math is fun, and it gave them a concrete, tangible experience to attach fractions to. The interview process with the adults was very valuable because it helped humble egos, and asked each individual to think about their own thinking more closely. For me, each interview was inspiring and exciting, and I am honored that I had the opportunity to do them. Each one exposed a different layer of meaning and use for the fraction orange, further shining light on its simultaneous complexity and simplicity. I left each interaction filled with a greater sense of awe than I had before.

Creating the fraction orange has been an invaluable experience. Having to create a manipulative to further mathematical ideas in my students required me to have a deep understanding of fractions myself, and it also gave me a sense of investment and motivation that I would not have in the material if I had not created my own tool. It expanded my mathematical thinking into discovery about the relationship we have to others' mathematical thinking and understandings, the relationship of the part to the whole, of the part to the part, and so on. The Maker experience sparked so many new ideas for me and the people who I interacted with, as it became about the relationships we have to our own understandings, and a call to put those understandings into question and move forward with a sense of inquiry, possibility, and wonder.

References

Baroody, A. J. (1987). *Children's mathematical thinking: A developmental framework for preschool, primary, and special education teachers*. New York, NY, US: Teachers College Press.

Empson, S. B., & Levi, L. (2011). *Extending children's mathematics: Fractions and decimals*. Portsmouth, NH: Heinemann.

Kimball, M., & Smith, N. (2013, October 28). The myth of 'I'm bad at math'. *The Atlantic*.

Lockhart, P. (2009). *A mathematician's lament*. New York, NY: Bellevue Literary Press.

Van de Walle, A., Karp, K., & Bay-Williams, J. M. (2013). *Elementary and middle school mathematics: Teaching developmentally* (8th ed.). NY, NY: Pearson.