EXPLORING THE INTERWOVEN DISCOURSES ASSOCIATED WITH LEARNING TO TEACH MATHEMATICS IN A MAKING CONTEXT

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Abstract

In this paper, we aim to explore how prospective elementary mathematics teachers (PMTs) learn to teach mathematics through their engagement in a pedagogically informative Making experience. Grounded in a commognitive perspective, we define learning to teach mathematics as changes in any of these four discourse activities: mathematizing and identifying (Heyd-Metzuyanim & Sfard, 2012), in addition to pedagogy and designing. We present our analysis of one PMT's discourse activity, exposing Making as an effective venue for provoking all four discourses and revealing their intertwined nature, further illustrating that one's identity is as central to learning to teach mathematics as is their learning of mathematics, pedagogy, and design. We conclude with a discussion of the implications of these findings for the research and practice of teacher education.

Keywords: Teacher Knowledge; Teacher Education - Preservice; Affect, Emotion, Beliefs, and Attitudes; Technology

Teacher knowledge literature continues to evolve, with recent conceptualizations building on previous characterizations of distinctive knowledge domains in order to promote a wider focus on their integration (Scheiner, Montes, Godino, Carrillo, & Pino-Fan, 2019). In the current study, we adopt this perspective by viewing *teachers as learners* and foregrounding their identities (Sfard & Prusak, 2005) in order to recognize what affective, interpersonal, and social matters can bring to this conversation. That is, by honoring the interrelationship between the learning of mathematics and the learners themselves, we hope to move beyond the "static, explicit and objective" (Scheiner, et al. 2019, p. 161) outlooks on knowledge to recognize the blended nature of knowing (Scheiner, 2015). And because our teachers are designing manipulatives to share with children with the intention of promoting their mathematical learning, the promise of this approach is suggested by the proposition that teachers' "invention[s] of 'objects-to-think-with'... [offer] the possibility for personal identification" (Papert, 1980, p. 11).

Adopting a communicational perspective on learning (Sfard, 2008), our objective is to explore the premise that learning to teach mathematics can be seen as changes in discursive activities that include narratives about mathematics and identity. The following question guides this research: *As practicing and prospective elementary mathematics teachers Make new manipulatives and corresponding tasks to support the teaching and learning of mathematics, what might their discourses reveal about the epistemology of learning to teach mathematics?*

Theoretical Framework

Our theoretical framing is organized around the learning theories of *commognition* and *constructionism*. Commognition encompasses both interpersonal "communication" and individual "cognition" (Sfard, 2007, p. 570). Discourse, with its affective and social aspects, is central to commognition, and learning is seen through changes in discourse (Heyd-Metzuyanim & Sfard, 2012). The constructionist perspective adds a dimension of participation in a discourse community with a view

toward the learning that can happen during the process of making a shareable object (Harel & Papert, 1991).

Heyd-Metzuyanim & Sfard (2012) take a commognitive perspective to frame mathematics learning as the interplay between talking about mathematical objects (*mathematizing*) and talking about participants of the discourse (*identifying*). Sfard (2008) defines discourse as "a special type of communication made distinct by its repertoire of admissible actions and the way these actions are paired with (re-)actions" (p. 297). Discourse can include speech, gestures, and visual mediators (e.g., graphs, symbols, manipulatives) (Sfard, 2008). From there, identity is viewed as a collection of "narratives about individuals that are reifying, endorsable, and significant" (Sfard and Prusak, 2015, p. 16), and identity discourse is viewed as integral to the learning of mathematics [see also Graven & Heyd-Metzuyanim (2019)]. We supplement mathematizing and identifying with two additional forms of discourse that also may be relevant to the learning of mathematics: *pedagogy* (narratives about teaching and learning) and *designing* (narratives about design decisions). Thus, this framework provides us with a lens through which to study how the process of making a manipulative can provoke the four discourse activities of mathematics, pedagogy, design and identity, and help us to see the intertwined nature of a teacher's learning.

Methodology

This project is part of a larger study that aims to test and refine the hypothesis that a pedagogically genuine, open-ended, and iterative design experience centered on the Making and sharing of a physical manipulative for mathematics learning would be formative for the development of practicing and prospective elementary mathematics teachers' (PMTs') inquiry-oriented pedagogy. That study took place in the spring of 2019 in a graduate-level mathematics course for PMTs at a mid-sized university in the northeastern United States. Thirteen participating students comprised ten groups (yielding ten projects). The PMTs were tasked with designing and 3D printing a manipulative that would be shared with a child to support their meaningful learning of mathematics. Written assignments provided autobiographical information of the PMTs' experiences as mathematics students, as well as reflections on clinical interviews they conducted throughout the semester. Snapshots of the PMTs designs in progress are included in the data, as are the physical "printouts" of their manipulatives and video recordings of the course's design sessions.

For this project, we took an exploratory case study approach (Yin, 2009) that focuses on "Moira," a PMT whose initial design was a tool intended to simulate the "keep change flip" algorithm for fraction division. She thought this tool would make fraction division meaningful by providing a concrete representation in which a child could *physically* "keep" the dividend, "change" the division symbol, and "flip" the divisor. However, the course's teacher educator pushed back on Moira's idea by asking her, "When dividing fractions, *why* do you flip the second fraction and multiply?" In re-action to this prompt, Moira becomes intent on figuring out "why we flip the second," a move that signals a change in her mathematical discourse. In a subsequent session, we noticed that she deviates from this intention, opting instead for a new fraction tool design that could support meaningful comparisons of fractions with a broader age-range of students. Effectively, her new design takes familiar fraction strips and connects them end to end to turn them into eight partitioned rings that can be stacked vertically on a cylindrical pedestal.

We chose Moira as an exploratory case because the change in her mathematical discourse constitutes learning, but we also sought to understand this learning through the lenses of the other discourses. Accordingly, we invited her back after the course ended for a voluntary, follow-up, semi-structured and task-based interview (Ginsburg, 1997). In addition to helping us understand Moira's rationale for abandoning her earlier fraction division design, we viewed the manipulative she had made for fraction comparison as an instance of her design discourse and sought to use it to assess her understanding of fraction division. This interview was video-recorded and added to the corpus of Moira's data, along with written artifacts from the interview. That data was then analyzed through the conceptual lenses of the four discourse activities: identifying, mathematizing, pedagogy, and designing.

Results

In this section, we present two central results from the follow-up interview. The first result concerns Moira's decision to change her tool design, and our analysis of this choice through the discourses of mathematizing [M], Pedagogy [P], Designing [D], and Identifying [I]. Moira reflects, "I wanted to make something that could be interpreted in many different ways [*M/P/D*], that wasn't something that I was just forcing them to, like, all right, you have to use it this way. I wanted it to be able to be manipulated [*M/P/D/I*]." As she considers her initial "keep, change, flip" tool, she articulates, "You basically were just, like, flipping the fraction upside down in my initial tool and ... it was just not useful [*M/P/D*] ... So I decided to switch to comparing fractions and then I came up with this [fraction comparison tool] [*M/D/I*]."

These reflections reveal how Moira's initial decision to abandon her fraction division design is not just about mathematizing, but also about identifying: as a teacher, it is important to her that her students have the opportunity to develop their own ways of thinking about fractions with a tool that can be used in a variety of ways. Moira acknowledged that the pedagogy promoted by the instructor in the classroom was also part of her decision to change her design:

> *Moira:* Well, [the change of design] was because we were talking and you [the teacher educator] said, "you're just teaching them how to – you're just giving them a way to solve the problem." And I realized, you're right ... It wasn't helping them learn how to do a problem [*M/P/D/I*].

By switching to a design for comparing fractions, Moira can participate in the discourse endorsed in the teacher education classroom and honor the teacher she wants to be.

A second result related to Moira's learning emerges from the interviewers' awareness that her current tool *could* be used to make sense of fraction division and a question about whether Moira realizes this capability in her tool. The interviewers ask her about this possibility, prompting Moira's in-the-moment reflections: "½ divided by 2. ½, this divides it into two equal parts, and I know this equals fourths, so this is ¼" [*M/I*]. Then, in investigating 1 divided by 1/3, Moira takes the 1 and 1/3 ring, guesses the answer is 3, and says, "I know I can do it, and I'm seeing it, but I don't know how to describe it" [*M/I*]. Moira is using her tool to make sense of this problem when the interviewers prompt her to explain whole number division (e.g., 6 divided by 3). As Moira reasons through whole number examples [*M*], she exclaims, "Oh! So, so, if I am dividing 1 by ⅓, there are three thirds in 1, so it's 3! Yes! You can do division with these … Wow! Fractions make so much sense now" [*M/I*].

Although Moira's reflections on the ½ and (later) ⅓ examples seem the same, the shift from her use of a partitive conception of division to a measurement one gives her sought-after language to "describe" her tool's utility in her understanding of fraction division. Moira's mathematical discovery is intertwined with an expression that reveals how emotionally invested she is in this realization. The moment culminates in self-reflection: "Honestly, I'm so impressed with myself [*I*]. I did not think that it had this capability. I thought it was only for comparing fractions [*M/P/D/I*]. So we've learned something today, haven't we all…" [*I*].

As she uses her tool to think through fraction ideas [*M*], Moira comes to recognize its potential not only for her own learning, but also for teaching fraction division in a way that aligns with her identity as a teacher [*P/I*]. Moira's body language and energy substantiate her enthusiasm for this discovery. Finally, the whole experience leads her to identify herself as part of a community of learners who can struggle and reason as part of a sense-making process.

Conclusion and Implications

As in a woven tapestry, learning to teach mathematics weaves together four threads or discourses that are unique to a PMT's discursive experiences and particular to a learning community where inquiry pedagogy is promoted. In this sense, to characterize Moira's learning to teach mathematics as a complex structure of discursive activities interwoven in dialectical unity is to illuminate the brilliance of a tapestry threaded by what she wants to teach (mathematizing), how she wants to teach it (pedagogy), decisions about what resources to make available (designing), and the kind of teacher she wants to be (identifying).

Zooming in on that tapestry might provide a view on a single thread of Moira's understanding of fraction division, but focusing on a single thread obscures the others with which it is interwoven. Collectively, these threads contribute to a more intellectually honest depiction of the "organic whole" (Scheiner, 2019, p. 165) that is learning to teach mathematics.

This project set out to explore the proposition that learning to teach mathematics can be credibly conceived as changes in mathematizing, identifying, pedagogy, and designing discourses. Our analysis of data related to Moira's experiences making a physical manipulative for sharing with a child reveals how her experiences provoked all four discursive activities, and revealed the intertwined nature of these discourses. This finding resonates with a view of mathematics teacher learning that emphasizes the blending and transformation of constituent knowledge domains into emergent knowing. It also resonates with an acknowledgment of the complex dynamics of mathematics teacher knowledge in action (Scheiner, 2019).

Our study also establishes that identity is as central to learning to teach mathematics as is the learning of mathematics, pedagogy, and design. The ensuing changes of discourse have revealed that although sometimes viewed as distinct, teacher learning domains are inherently connected to a PMT's identity. In light of research by Pratt and Noss (2010), its centrality can be understood in the context of a design project carried out in a Maker community where PMTs were engaged in creative activity, leveraging their personal experiences and invoking personal design decisions, reflections, and articulations. All in all, implications of this finding speak to the potential of interdisciplinary experiences like the design experience as venues for the meaningful learning of learning to teach mathematics within teacher preparation coursework.

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