Final Reflection

By Jade

**all names and identifiers have been masked/changed to retain anonymity*

Introduction

The manipulative I have made is a 3D version of fraction strips. Each strip was made to be a rectangular/square piece that slides into individual pegs making it sturdier and easier to play with. I've also made it so that the fraction blocks stack vertically (rather than lining up horizontally), to indicate height as value and amount. I wanted to build something to help the student to visualize and deepen student understanding as they explored fraction relationships. What was most important to me was having all the fractions mounted on one platform with the 1 (whole) always being visible, so that the student could begin to grasp how all the smaller parts can equate and compare to the whole.

Key Design Decisions

Due to printing constraints, we could only print 3 pegs. It worked out ok, but I originally wanted all the fraction blocks to be visible to give a clearer more concrete visual about how the equal parts were broken up and labeled. The size, proportions and measurements were very important to the manipulative because each fraction block had to be measured perfectly to its fraction equivalent, as well as to fit into the pegs for easy sliding. The colors didn't matter much, it would have been an addition to see the different colors of the fraction blocks as being separate, as well as aesthetically pleasing, but it did not affect how the manipulative worked.

The original test print worked out great and the blocks fit into the pegs. No changes were made, but the final print of the Tinkercad did not print correctly and the blocks no longer fit the pegs. A few other issues that came about during final print: the holes on the inside of the blocks were still filled in with the 3D material (which defeats the point of being able to slide it into the peg). Some of the fractions imprinted on to the blocks were not sanded which made it more difficult to read the numbers. I did not change my mind about how to design the parts of my tool except to print earlier

Mathematical Task Statements – With and Without Manipulative

Similarly, to the children in the *5 Indicators of Decimal Understanding* reading, I think a child may misunderstand the numbers in the numerator and denominator as being separate values and not distinct from whole numbers. Students may struggle with fractions because they "overgeneralize their whole-number knowledge" (Van DeWalle, p. 320). For example, Gemma believed $\frac{1}{10}$ was larger than $\frac{1}{2}$ because the 10 in the denominator as a whole number is larger than the 2. Students may apply whole number ideas to fractions, disregarding what an actual fraction means (part of a whole). I think having visual representation of the fractions as pieces compared to the whole (1) can help a child understand that ten of the $\frac{1}{10}$ pieces make up 1 whole piece, so 1 part ($\frac{1}{10}$) would be smaller than the whole piece (1), or as in the example above, smaller than the $\frac{1}{2}$ piece. With the use of the manipulative, I am hoping to focus on three tasks: parts to a whole, fraction operations and equivalents in order to help the student conceptualize fractions.

The first task I presented was to help construct the student's idea of "fractional parts of the whole – the parts that result when the whole has been partitioned into equal-sized portions" (Van de Walle, p. 292). The question was, "Gemma has one chocolate bar. She wants to split it equally between herself, her mother, and her sister. How can she split it so all 3 people have equal pieces? Which fraction would show that each person received equal pieces?" By representing that the chocolate bar is one piece (the whole), I want her to see which peg can be split into 3 pieces evenly. My hopes are that through this exercise, the student will understand parts of a whole must be partitioned into equal sized pieces, as they see how the whole is broken up into 3 pieces (the denominator) and the number of pieces indicates the numerator ($\frac{1}{3}$). Without the manipulative, a student may not understand what the numerator and denominator actually represent.

The second task was to demonstrate how many parts make up the whole, using addition and subtraction operations. The question was, "Penny's school is one mile away from her house. She walks $\frac{3}{5}$ of the 1 mile to get to her corner, and then turns left and walks $\frac{1}{5}$ of the 1 mile. How much did she walk in total?" I decided to keep the denominators the same value so she could see by stacking the number of pieces means the numerator gets added while the denominator remains the same. After discussing and seeing if she understands this concept, the second part to this question to display parts to a whole was, "How much more does Penny have to walk to get to school?" Since the 1 (whole) will be stacked on the end, I want to deepen her understanding that 5 pieces of the $\frac{1}{5}$ part equal to the whole (1).

Without the manipulative, a student may apply addition operations to both the numerator and the denominator. For example, seen in the last interview, when adding $\frac{5}{12}$ and $\frac{3}{12}$, Gemma originally wrote the two fractions in a vertical equation and answered, "824" (written exactly as is, without the fraction bar). With the manipulative, by seeing the pieces all stacked on top of each other, a student can directly count that there are 4 pieces in total, which would be the numerator. By adding the second part to the question, how many more to make the whole, can deepen their understanding that there are 5 pieces in total, which make up the value of the denominator.

In extending the second task, the third problem was using subtraction/addition operations with comparing a part to the whole. “Jaden is $\frac{1}{4}$ done with his homework. How much more does he have left to do?” There will be no other fractions to compare or add/subtract. Instead, this fraction/part would have to be compared to the whole. Without the manipulative, a student may not know where to begin, which numbers would need to be subtracted. With the manipulative, a student can start stacking the number of parts using the $\frac{1}{4}$ pieces and conceptualize that 3 more pieces or $\frac{3}{4}$, make up the whole.

The last task is comparing fractions: $\frac{2}{3}$, $\frac{3}{6}$, and $\frac{5}{8}$. I purposely chose these fractions for students who may apply whole number values and believe the larger whole number value in the denominator would mean it was the largest. By using these fractions, there is no pattern where a “rule” could be applied – the order from smallest to largest would be: $\frac{3}{6}$, $\frac{5}{8}$, $\frac{2}{3}$. I am hoping by using the manipulative to stack the fractions next to each other, a student will be able to see which ones are taller or smaller and reason that that means the fraction value as being more or less, rather than without the manipulative where they compare the values of the numerator or denominator to get their answer. The question is, “Jack and his two friends each had the same size pizzas for lunch. Jack ate $\frac{5}{8}$ of his pizza. Judy ate $\frac{2}{3}$ of her pizza. And Sam ate $\frac{3}{6}$ of his pizza. Who ate the most amount of pizza? Who ate the least?”

The last task in my problem set is understanding equivalent fractions. Without the manipulative, students may not reason that the same amount or value, could be represented with different numbers or fractional values. I would like to challenge that concept with using the manipulative to be able to understand that there are multiple ways of showing or describing the same amount by using the different sized pieces. They may have different "names" or values, but hopefully by stacking the parts to equal lengths, a student will begin to see that the same values can have different representations.

Findings

Phase 1, Manipulative Play

During initial play, Gemma moves the block around for a little bit, looking at the pieces and sliding them off the pegs. After seeing the whole piece (1) and the $\frac{1}{2}$ piece, she says she remembers from 1st and 2nd grade doing something similar with circles. She explains that she learned that there are "parts" and a "whole". She tries to show using the 3 pieces (1 piece, and two $\frac{1}{2}$ pieces), manipulating the way it is laid out, that you use the two parts trying to find the whole. In one example, she places the two ½ pieces in a "V" shape coming from the 1 piece.

Phase 2, Manipulative Play

I first asked what she thinks we'll be doing with the manipulative. She replied, "Solving problems. Word problems?" I explain we'll be doing both those things with fractions. I explain how you can use the manipulative to "see" the equal parts of a whole, "see" which fractions may equal the same amount using the height of each combination. Before the problem set, we lay out all the pieces the way it would have been on the original design of the manipulative, with the pieces stacked on top of each other in order, but on the table instead of on the pegs. We find that we are missing one of the fraction blocks ($\frac{1}{5}$). After stacking a few fraction blocks, I further explain the equal heights mean equal amounts, which all equal to the whole (1).

Phase 3, Pre-Task Exploration

To show her how to use the tool, I ask, “if I want to find fractions equal to the ½, what could I do?” She chose to use the $\frac{1}{6}$ blocks and places three of them on top of the $\frac{1}{2}$ to the same height as the “whole” piece. I show her she could also put it in the peg next to the $\frac{1}{2}$ and make sure the heights are the same. She says she understands how the manipulative works, so we continue to the task set.

Phase 4, Task Exploration

To begin, I first ask, “What do you think 3 over 4 might look like?” She immediately finds the $\frac{1}{4}$ blocks and slides three on to one peg. So, I continue, “What do you think the 3 stands for?” She replies that there are 3 of them. Then, “What do you think the 4 stands for?” She seems confused, so I write the fraction out on a piece of paper to show her the fraction (I should have done that before verbally asking). She replies, that “there are not exactly of them. It’s not exactly 3 out of 3.” I ask her to show me what $\frac{3}{3}$ would look like. (She has difficulty during this because the manipulate is not laid out as intended. She looks for the pieces). After finding the $\frac{1}{3}$ blocks, she puts 3 on top of each other on the peg. She says, “this is one whole”. When we go back to the $\frac{3}{4}$ fraction, and I ask again, “what do you think the 4 stands for?”, she struggles. Either she doesn’t have the vocabulary or doesn’t understand how to explain it. I figure we will find out through the tasks.

Task 1. Gemma has one chocolate bar. She wants to split it equally between herself, her mother, and her sister. How can she split it so all 3 people have equal pieces? Which fraction would show that each person received equal pieces?

She quickly smiles and picks three of the $\frac{1}{3}$ blocks and stacks them on the peg (next to the whole block which we kept on the platform). When I ask how she quickly solved that, she replies, "because they are equal length, there are three people, and there are three of them." It seems she understands equal parts to a whole. I think this was a good task to start with, it was easy to relate to and she could feel confident in playing with the manipulative.

Task 2. Penny's school is one mile away from her house. She walks $\frac{3}{5}$ of the 1 mile to get to her corner, and then turns left and walks $\frac{1}{5}$ of the 1 mile. How much did she walk in total? How much more does she have to walk to get to 1 mile?

Gemma begins by getting three of the $\frac{1}{5}$ blocks. She says she understands that Penny walks "3 out of 5". She slides them on, and after looking at the problem, she adds one more $\frac{1}{5}$ block and answers, "it would be 4 out of 5 miles." She says she can see there are four pieces so that is the answer. She was able to use visual confirmation and math facts to solve the problem. I ask the second part of the problem, "How much more does she have to walk to get to 1 mile?" She says, "One more $\frac{1}{5}$." When asked how she got that, she says, "If I had one more piece, I would put it on top and it would equal the same amount" (indicating the whole). She used math facts/memory and could quickly answer that 1 more block would equal $\frac{5}{5}$. She seems to have an understanding of what the numerator represents.

Task 3. Jaden is $\frac{1}{4}$ done with his homework. How much more does he have left to do?

Right away, Gemma puts three of the $\frac{1}{4}$ blocks on a peg. She says, “This is what he would need. 3 more pages of homework to do. I know that he has 4 out of 4 pages of homework to do, so if he only did 1, then I know he has 3 more left to do.” I’m not sure if she counted up or down from the 4 of the denominator, but she was able to quickly do the math in her head and show the solution using the blocks.

Task 4. Jack and his two friends each had the same size pizzas for lunch. Jack ate $\frac{5}{8}$ of his pizza. Judy ate $\frac{2}{3}$ of her pizza. And Sam ate $\frac{3}{6}$ of his pizza. Who ate the most amount of pizza? Who ate the least?

Considering she is doing really well with playing with the manipulative, before we go into Task 4., I ask her to show me what $\frac{5}{8}$ would look like using the blocks and manipulative. Gemma takes five of the $\frac{1}{5}$ blocks ($\frac{5}{5}$) and slides them on to one peg. Then eight of the $\frac{1}{8}$ pieces ($\frac{8}{8}$) and slides it on the peg next to it. She explains that there would be 5 on the top and 8 on the bottom. I think she understands what a fraction is supposed to look like but doesn’t fully grasp the concept of it being parts to a whole with using the manipulative.

I go on to ask what $\frac{2}{3}$ would look like. She does the same thing, puts two of the $\frac{1}{2}$ blocks on one peg, then three of the $\frac{1}{3}$ blocks to the peg next to it. (Note, that there was also a lot of distraction at this time – people were coming in and out of the house, dogs were barking, etc.). After it settles, I go back to Task 3., and ask her to show me $\frac{1}{4}$. She takes only one of the $\frac{1}{4}$ blocks and slides it on to the peg (got it correct). So, I ask to show me $\frac{2}{4}$, and she adds another $\frac{1}{4}$ block on top of it. When I ask her to show me $\frac{2}{3}$ again, she reverts and adds two of the $\frac{1}{2}$ blocks (to represent the 2 in the numerator) and three of the $\frac{1}{3}$ blocks on the peg next to it (to indicate the 3 of the denominator). At this point, I can only guess that she understands how to show it correctly when the numerator is a 1, but when the numerator is higher, she might be getting confused. To be honest, I wasn't sure what was going on, so I decided to go onto the task.

After reading the problem, and before using the manipulative, she answers, "I think Jack ate the most ($\frac{5}{8}$)". As I mentioned above in the mathematical statement – without the manipulative, she may be misunderstanding the numbers in the numerator and denominator as being separate values and not distinct from whole numbers. While using the manipulative, I ask her to show me what $\frac{5}{8}$ would look like, "using the $\frac{1}{8}$ blocks". She stacks five of the $\frac{1}{8}$ blocks correctly. Gemma continues to slide the blocks on the pegs according to the fractions in the task, $\frac{2}{3}$ and $\frac{3}{6}$. When we go back to reading the problem, even while looking at the manipulative, she answers, "Jack ($\frac{5}{8}$)". When I ask her to explain her reasoning, she calls out the numerators and says that the 5 is the biggest number, the $\frac{2}{3}$ is the smallest because of the 2 in the numerator. The values are still separate amounts for her.

I decide to flip the manipulative around, so the imprinted fractions don't show, and ask her again which one has the "most". When using just visual confirmation, she points to the highest peg ($\frac{2}{3}$). I flip it around to show her which fraction amount these indicate. I try to probe and ask what she thinks the numerator means again. She doesn't have the vocabulary yet, but understands its only part of the whole amount. I think seeing the numbers as separate values, written on paper makes her automatically assume that the higher numerator is the greater amount. But when only looking at the blocks, she can clearly see which one is taller in height (which is to represent the amount).

To conclude, I decide to go into fraction equivalents. I put one of the $\frac{1}{2}$ blocks on a peg and ask her to show which fractions might equal the same "amount" or "height". Gemma begins to play around using only visual estimates. The first one she picks is the $\frac{1}{4}$ block and places two of them on the peg next to the $\frac{1}{2}$ block. Then she starts playing around by picking different fraction blocks and stacking them on one peg. She picks $\frac{1}{8}$; $\frac{1}{5}$; $\frac{1}{6}$ (it was very close!)

Reflections

I learned I have to be more patient when doing this. I found it hard not to "explain" how to solve the tasks or to not "correct" the student. I understand this was to have the child play and show what we could do with the manipulative, but while doing the tasks I found it difficult to scaffold and try not to explain it for her understanding. I would really like to try this again with the full manipulative (all the pegs and pieces). I think if we continued to play with it, while all the fractions were visible, she could more easily discern the differences in the heights and connect that to the actual fraction amounts. During the interview, I was also feeling anxious about the video not recording or losing the feed (as I did the first interview). I think I hurried through some of the tasks this time and felt rushed. But now that I have some practice, I think having the right equipment and resources for video editing, that I could spend more time letting the child "play" more, rather than trying to get the tasks completed.

I really do feel that Gemma had fun doing this. She was also a little nervous but I could see she was eager to learn and be challenged. She was very confident in her answers while also being receptive to the "corrections" or explanations. As a teacher, I found this valuable because

I'm learning new creative ways for students to learn. Not all students learn the same way, and having a better understanding of how they learn, what they are interested in, can help to further tailor instruction. It was fun to see what would come out of this and be able to reflect on how I could continue to learn from a student, how to interact with students, and how to scaffold their learning for deeper and more meaningful learning.

Conclusion

In the Project Rationale, I said the ultimate goal for the project to be successful would mean that the student has a new way of thinking about fractions. I also wanted the student to be able to see fractions as different from whole numbers. I think Gemma was able to play around with fractions easily and was starting to "see" that the numbers of the numerator and denominator represent different values. I said to be "unsuccessful in terms of learning" would be where the student really ends up disliking fractions and where the manipulative does not help them to connect their ideas with the visual model representation. For the first part of that statement, I can say that Gemma enjoyed using the manipulative. Even after we finished, she said she wanted more tasks. I think she was definitely learning and feeling challenged, which was boosting her confidence in continuing. I think if I had more tasks, she would have been able to start conceptualizing that the numerator represents how many equal parts of the denominator.

Overall, I think Gemma was beginning to conceptualize and reason that these blocks together make up one whole and was able to see the separate blocks as parts of a whole. Although she was struggling towards the end (with the distractions), I think she was able to connect the manipulative with the given fractional values ($\frac{1}{2}$ = one of the $\frac{1}{2}$ blocks or two of the $\frac{1}{4}$ blocks, etc.) and she was getting a greater emphasis of number sense. I think seeing and counting the fractional parts as being multiple and separate parts helped her to compare it to the whole to understand the relationship between the pieces (numerator) and the whole (denominator). In the end, we both had a good time and I loved that she wanted to keep going. I'm just happy she didn't end up disliking fractions but said she really enjoyed it too!

References

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