Project Rationale

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**all names and identifiers have been masked/changed to retain anonymity*

Description of Manipulative

The manipulative I have decided to make is a 3D version of fraction strips. Each strip is now a rectangular piece that will slide into individual pegs making it sturdier and easier to play with and move around. The fraction strips are great on their own, but they slide which makes it difficult to really visualize the lengths of the fraction parts if they are off alignment. I've also made it so that the fraction strips stack vertically (rather than lining up horizontally), to indicate height and value. As, "manipulatives are an integral part of instructional tools for deepening student understanding" (Van de Walle, 2013, p. 142), I wanted to build something to help the student to visualize and explore fraction relationships. What is most important to me is that by having all the fractions mounted on one platform with the 1 (whole) always being visible, that the student will begin to grasp how all the smaller parts can equate and compare to the whole.

I've been teetering on the idea of adding the decimal and percent equivalents of the fractions on the other faces of each piece, but I'm not sure if this might make it more confusing or be helpful for her later. I believe having a strong understanding of fraction concepts will also deepen students' understanding when learning about decimals and percent concepts later, so I am still on the fence about this. I didn't have to modify anything to my design, but I decided to modify the problem set with what I learned from the last interview.

Mathematical Tasks – With and Without Manipulative

Similarly to the children in the *5 Indicators of Decimal Understanding* reading, I think a child may misunderstand the numbers in the numerator and denominator as being separate values and not distinct from whole numbers. Students may struggle with fractions because they "overgeneralize their whole-number knowledge" (Van DeWalle, p. 320). For example, Gigi believed $\frac{1}{10}$ was larger than $\frac{1}{2}$ because the 10 in the denominator as a whole number is larger than the 2. Students may apply whole number ideas to fractions, disregarding what an actual fraction means (part of a whole). I think having visual representation of the fractions as pieces compared to the whole (1) can help a child understand that ten of the $\frac{1}{10}$ pieces make up 1 whole piece, so 1 part ($\frac{1}{10}$) would be smaller than the whole piece (1), or as in the example above, smaller than the $\frac{1}{2}$ piece. With the use of the manipulative, I am hoping to focus on three tasks: parts to a whole, fraction operations and equivalents in order to help the student conceptualize fractions.

The first task I would like to present is to help construct the student's idea of "fractional parts of the whole – the parts that result when the whole has been partitioned into equal-sized portions" (Van de Walle, p. 292). The question is, "Gigi has one chocolate bar. She wants to split it equally between herself, her mother, and her sister. How can she split it so all 3 people have equal pieces? Which fraction would show that each person received equal pieces?" By representing that the chocolate bar is one piece (the whole), I want her to see which peg can be split into 3 pieces evenly. My hopes are that through this exercise, the student will understand parts of a whole must be partitioned into equal sized pieces, as they see how the whole is broken up into 3 pieces (the denominator) and the number of pieces indicates the numerator ($\frac{1}{3}$). Without the manipulative, a student may not understand what the numerator and denominator actually represent.

The second task is to demonstrate how many parts make up the whole, using addition and subtraction operations. The first question is, “Penny’s school is 1 mile away from her house. She walks $\frac{3}{10}$ of a mile and then $\frac{5}{10}$ of a mile. How many miles did she walk in total?” I decided to keep the denominators the same value so she can see by stacking the number of pieces means the numerator gets added while the denominator remains the same. After discussing and seeing if she understands this concept, the second part to this question to display parts to a whole is, “How much more does Penny have to walk to get to school?” Since the 1 (whole) will be stacked on the end, I want to deepen her understanding that 10 pieces of the $\frac{1}{10}$ part equal to the whole (1).

Without the manipulative, a student may apply addition operations to both the numerator and the denominator. For example, seen in the last interview, when adding $\frac{5}{12}$ and $\frac{3}{12}$, Gigi originally wrote the two fractions in a vertical equation and answered, “8/24”. With the manipulative, by seeing the pieces all stacked on top of each other, a student can direct count that there are 8 pieces in total, which would be the numerator. By adding the second part to the question, how many more to make the whole, can deepen their understanding that there are 12 pieces in total, which make up the value of the denominator.

In extending the second task, the third problem would be using subtraction operations with comparing a part to the whole. “Jaden is $\frac{1}{4}$ done with his homework. How much more does he have left to do?” There will be no other fractions to compare or add/subtract. Instead, this fraction/part would have to be compared to the whole. Without the manipulative, a student may not know where to begin, which numbers would need to be subtracted. With the manipulative, a student can start stacking the number of parts using the $\frac{1}{4}$ pieces and conceptualize that 3 more pieces or $\frac{3}{4}$, make up the whole.

The third task is comparing fractions: $\frac{2}{3}$, $\frac{3}{6}$, and $\frac{5}{8}$. I purposely chose these fractions for students who may apply whole number values and believe the larger whole number value in the denominator would mean it was the largest. By using these fractions, there is no pattern where a "rule" could be applied – the order from smallest to largest would be: $\frac{3}{6}, \frac{5}{8}, \frac{2}{3}$. I am hoping by using the manipulative to stack the fractions next to each other, a student will be able to see which ones are taller or smaller and reason that that means the fraction value as being more or less, rather than without the manipulative where they compare the values of the numerator or denominator to get their answer. The question is, "Jack and his two friends each had pizza for lunch. Jack ate $\frac{5}{8}$ of his pizza. Judy ate $\frac{2}{3}$ of her pizza. And Sam ate $\frac{3}{6}$ of his pizza. Who ate the most amount of pizza? Who ate the least?"

The last task in my problem set is understanding equivalent fractions. Without the manipulative, students may not reason that the same amount or value, could be represented with different numbers or fractional values. I would like to challenge that concept with using the manipulative to be able to understand that there are multiple ways of showing or describing the same amount by using the different sized pieces. They may have different "names" or values, but hopefully by stacking the parts to equal lengths, a student will begin to see that the same values can have different representations.

Further Learning with Manipulative

I think by playing with the manipulative and conceptualizing fractions using a model, this will be helpful for later concepts such as computational estimation, where the student can see which ones are closest to the 0, $\frac{1}{2}$, or 1. Even later, a student can reference this manipulative in thinking about decimals where a student may also apply whole number facts and applications. It will help to see how the smaller the fraction, the smaller the decimal value is as well.

Understanding of Mathematics and Learning

Van De Walle text explores how math rules cannot be taught, but rather be "a creation of each student's own thought process through many experiences" (p. 339). As we've seen in the YouTube videos, children use intuitive skills to solve math problems. It's not about teaching rules to be memorized, but about showing them different ways to conceptualize their understanding of what is being asked and deepen their understanding of how and why, rather than being answered, "It's the rule". I would like to build upon their intuitive reasoning and adding the manipulative to reflect how children can conceptualize fractions concepts. I think seeing and counting the fractional parts as being multiple and separate parts can help a child compare it to the whole to understand the relationship between the pieces (numerator) and the whole (denominator).

Students need to be able to understand the deeper meaning to conceptualize, in order to truly "learn" it. Rules are effective in getting correct answers, but there is no real reasoning or thought about the value. I want to be able to give the student the opportunity to think about the relative sizes of various fraction before learning rules, so that she can develop familiarity with or number sense about fraction size. "The goal is reflective thought, not the memorization of an algorithmic method of choosing the correct answer" (Van DeWalle, p. 339).

Successful and Unsuccessful in Terms of Learning

The ultimate goal for the project to be successful would mean that the student has a new way of thinking about fractions. Whether that's through the tasks being given, or by her simply playing around with the manipulative, I would like for the student to be able to see fractions different from whole numbers. To be unsuccessful in terms of learning would be where the student really ends up disliking fractions and where the manipulative does not help them to connect their ideas with the visual model representation.

For the last interview, I'm thinking of first showing the student the manipulative without the fraction values being visible and asking her to play around with it. She will see that all the heights are the same but are broken up into different number of equal pieces. My hopes are that she can begin to reason that these pieces make up one whole, and once she sees the fractions, that the whole number rules do not apply for fractional values, but rather, sees it as parts of a whole. I would like that the student will be able to connect the visual with the given fractional values and procedures in manipulating them. I would like for the project to give her a greater emphasis of number sense and the actual meaning of fractions, rather than using repetitive memorization rules. Lastly, of course, that she has fun while playing with it!

References

Van de Walle, J. A., Karp, K., & Bay-Williams, J. (2013). *Elementary and Middle School Mathematics Teaching Developmentally.* Pearson: Upper Saddle River, NJ.

Cramer, K., Monson, D., Ahrendt, K. C., Wiley, B., & Wyberg, T. (Oct 2015). 5 Indicators of Decimal Understandings. *The National Council of Teachers of Mathematics, Inc.* 22(3), 186-195.