MTHM 577: Mathematics Education in the Elementary School

Making for Learning: Final Paper

The Design Case of No Más Caídas

Maya & Zoe | Spring 2019

"Rather than pushing children to think like adults, we might do better to remember that they are great learners and to try harder to be more like them." -Seymour Papert

INTRODUCTION

This 'Making for Learning' project, undertaken in our graduate course studying how elementary school children learn and do mathematics, presented us with what appeared a straightforward task: *use a 3D printer to make a tool for teaching children math*. Over the course of the semester, we have been a part of an ongoing conversation that has weaved together an intricate web of design choices. As the two of us, Maya and Zoe, made those choices together, we attempted to remain grounded in our simple pursuit of providing children something fun to count with. What it means to us for children to actually enjoy the act of counting, and ultimately learning, has guided the evolution of our manipulative from a tiny red stick-like object into the 3-foot-tall tower, what we call *No Más Caídas*, that it is today. We experienced two major forces that influenced our design process: the 3D printing technology, which often presented productive constraints, and the real-life reflections of our tool's users, which often introduced transformative opportunities. Both have shown us firsthand what the power of collaborative iteration blended with creativity can bring to the teaching of math.

DESIGN IDEA & RATIONALE

The intended mathematical idea behind our tool has been an emerging one throughout the semester, which is perhaps one of our most prominent early learnings from this project: that is, when we design for teaching and learning, our work is constantly evolving. The underlying purpose of our tool, however, has always been for students to love the process of doing and learning math. *No Más Caídas*

has unfolded as an invitation for children of mathematics to create, engage, and solve questions by providing them with a uniquely simple and organized way to count with concrete objects (marbles).

Our hope is that by providing children with opportunities to see, touch, and hear the simple objects they are counting as they fall into the tall tool, *No Más Caídas* entices all of its users to physically and completely experience what doing math can feel like. "Learning, if it is to be properly digested, involves the whole person. This is not just a peculiarity of childhood" (Pickard 1967). By opening up the freedom to play through counting, connections to learning can happen. After all, "As they play, the children are the ceaseless agents of their learning, about both the outer and inner world. The dynamic nature of their spontaneous interest and absorbed energy shows play to be a major time of discovery and adjustment to fresh information." (Pickard 1967). We can find children doing math by "counting as playing" across countries and cultures, with children in America playing counting games like hide-and-seek and hopscotch. Children in the Dominican Republic engage in counting games like *quien trajo más* (who brought more, which is like 'Simon Says,' but based on numbers) and *el que dice* _____ *sale!* where a number is picked, and whoever says it as they are counting is out. In the spirit of such games, we strive for *No Más Caídas* to elicit the everyday joy of children counting and thinking mathematically.

Zoe. As I (Zoe) drew the original sketch that expressed my idea for the first time to others, I was envisioning a sliding ruler as a representation vehicle for helping children develop the concept of multiplication as repeated addition. This idea was inspired by my case study student, Yasmine, a wide-eyed, beautifully brilliant 7-year-old girl who expressed a desire to learn multiplication in order to tackle third grade with mathematical confidence. Maya's initial idea was for an organizational tool to

hold katie cubes for counting, presenting a solution to a dilemma that Yasmine experienced during our problem solving interview. When working with numbers more than twenty, second grader Yasmine kept losing track of which katie cubes she was counting, and would need to restart her thinking again back from 1, hindering her progress in solving the problem presented. I only identified this as an opportunity to create a manipulative once Maya told me of the solution she was thinking of to help her student, Veronica. I became excited that our potential tool could be made into something that could help more than just one student.

Maya. Inicialmente, yo quería crear un instrumento que ayudara a Veronica a organizar mientras contaba. Ella se distraía, lo cual le causaba frustración. Matemáticas es más que sentirse frustrado, matemáticas para mi es la oportunidad de conocer más a fondo a un determinado espacio u objeto, entonces pensé en un artefacto que permitiera a Veronica disfrutar lo que hacía sin espacio a distracción, algo que organizara mientras contaba.

During our first in-class design session, we shared a discussion together that molded our two ideas into one: *let's put Zoe's sliding ruler onto Maya's counting tool*. While both of our designs were intrinsically simple--a counting tool and a sliding ruler--merging them together empowered new possibilities, as the ruler could promote counting by certain number intervals (i.e., counting by fives, which is the interval we ultimately decided to put onto the tool). We utilized our shared concerns on how the katie cubes kept slowing down and potentially confusing the counting and thinking process of our students as a design catalyst. That is, constructing our particular math tool in removing the worry around losing track of these "falling manipulatives" gave us something to create that had meaning for both of us and our students, and provided the name of our tool, *No Más Caídas* (which translates in

English to, "falls no more"). We chose marbles as the objects to be counted as a symbolic token of Maya's country, the Dominican Republic, and her childhood experiences there. In a country where learning resources are scarce, it is common for Dominican children to count with everyday and ordinary objects like rocks, stones, or beans, rather than with the fancy manipulatives that can be found in the American classrooms of schools in affluent neighborhoods. In addition to giving our tool a Spanish name, selecting marbles as the counting objects became a mark of Maya's Dominican culture.

The design behind our tool technology allows the child to stack the shiny, different colored marbles that they are working with directly on top of each other, creating an ever-growing compact tower of marbles held up by the tool. By placing each one on top of the other in the container as they count one at a time, each piece falls easily into place on top of the one before it. At this moment *No Más Caídas* is three tubes tall, where one tube is one foot in height each. We ended up settling on one foot per tube, merely for the sake of consistency with a regular metric ruler. When deciding how high to make our tool, we knew that the container could not be bigger than a foot and a half due to printer capabilities, so we decided to use peg inserts to make our container bigger than a foot and a half. The peg connection design of our technology (each tube has three pegs on its bottom and three open inserts on its top) allows for additional tubes to easily connect with each other, as the peg inserts go smoothly into the openings of the tube below it.

In order to serve a range of students' needs and ages, we printed the three tubes in total with the intention of Veronica, who is in preschool, using just the first tube (counting up to 20), and Yasmine, who is in third grade, using all three tubes (counting up to 60). This leaves open the possibility to only keep adding more and more tubes thanks to the way we designed the tool's structure, especially if the

base rests on the floor. It's exciting to us that students could use our tool to count to over 100+ with the creation of only five tubes, and the tool can potentially match their height. Our intended design was so that *No Más Caídas* can be as tall as we desire, since we can simply re-print copies of the tube over and over again with the 3D printer. This limitlessness of the height of the tool is an expression of what we believe learning math should feel like.

While we thought our manipulative could be a clear representation of *children's thinking,* our professor, Dr. Greenstein, remained unconvinced; what could we do to make it more than just a clear representation of *a collection of objects*? This pushed us to more distinctly define its intended purpose beyond acting as an organizational tool and search for a mathematical richness that we intuitively believed was embedded in our design. During our discussions with each other, we felt that it was there, just waiting to be uncovered during the process of designing. As we continued to iterate on our idea, we noticed that our tool can support a fundamental understanding of the nested nature of numbers; that is, the number 3 contains 1 and 2, it's not merely a number that is third in the list. *No Más Caídas* can help students make sense of number decomposition, as the ruler can help students recognize how the counting numbers are embedded in each other, allowing the student to construct knowledge that will be useful in breaking numbers apart (and together).

The first example of how we envisioned our tool to work mathematically is shown below, taken from our original project rationale: *The student adding 12 + 9 could take twelve objects at a time, put them into the container. Then, the student could take the remaining nine objects. Without having to count all twenty one objects in total, the student could use the ruler to see almost instantly what twelve objects against the ruler look like, and then what adding nine objects does - it lines up next to the*

number just one object past the four markings, which represents 20. Such an example is not unlike what we saw when we tried out our tool with Maya's daughter, Nashalie. Although we initially envisioned this tool as a student aide in adding, students can also use the tool for subtracting, multiplying and or dividing problems. *No Más Caídas* now also includes a base with six compartments, providing a space for students to sort the colors of the marbles and notice patterns. Additionally, *No Más Caídas* can be used as a measurement tool, which served as the basis for one of our tasks. As we begun to discover more mathematical uses for our tool, we realized what our tool was becoming went far beyond both of our original ideas.

Nuestro reto inicial era aprender a usar Tinkercad. Este programa tridimensional nos permitiria imprimir nuestra idea y a la vez era algo nuevo y sumamente necesario. Sin este, *No Más Caídas* se quedaría en simplemente palabras, Tinkercad nos daría la oportunidad de ver nuestras ideas realizadas, el ver a nuestras estudiantes (Yasmine and Veronica) contar, sumar, restar, multiplicar y dividir mientras usan nuestra herramienta. Después de haber navegado este programa tridimensional y con la ayuda de nuestro profesor, nos dimos cuenta de que no era tan complicado como lucía al inicio. Otro detalle que nos intimidaba era encontrar el material adecuado que permitiera visualizar y contar las canicas una vez que estas se encontraban dentro del cilindro. Este material debería ser lo suficiente claro, o sea, transparente, de manera que nuestras estudiantes pudieran tener un conteo preciso.

Por último, nos preocupaba la estructura de nuestra herramienta y el poco tiempo que teníamos para probarla, por lo que tuvimos que usarla con otra estudiante, mi hija Nashalie, la cual a diferencia de Verónica, estaba en preescolar, no en un jardín de infancia en el cual los estudiantes están más expuestos a problemas de matemáticas. En preescolar, Nashalie estaría aprendiendo a diferenciar entre

letras y números, más sin embargo, Ella fue capaz de trabajar con los números antes presentados a Verónica. *No Más Caídas* era más que un tubo para introducir canicas, necesitábamos suficientes tubos que abastecieran las necesidades tanto de Nashalie (preschool) como las de Yasmine (second grade). Nuestras estudiantes tenian diferentes necesidades, Nashalie era capaz de trabajar con números hasta 36, más sin embargo, debido a su grado escolar, Yasmine necesitaba más tubos que Nashalie, por lo que decidimos agregar conectores que permitieran trabajar con un sin número de tubos y por lo mismo trabajar con números más grandes.

KEY DESIGN DECISIONS

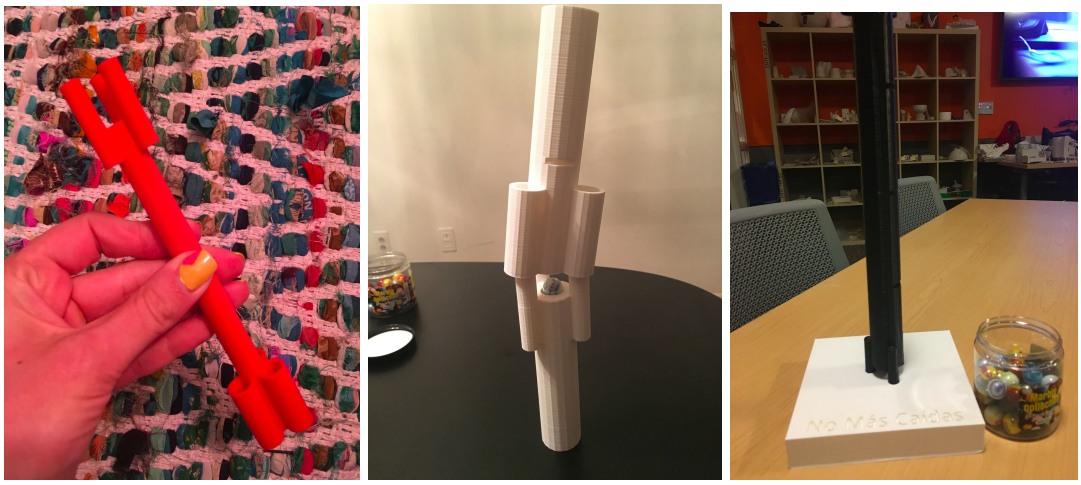
Much of our design process revolved around the discussion of addressing the following key questions:

- How is *No Más Caídas* all going to fit together?
- What does the ruler look like?
- How big should the marbles be?
- How tall should we make the container? How many tubes do we need to serve the mathematical needs of as many students as possible? How many numbers should *No Más Caídas* go up to?
- How are we going to get the marbles out?

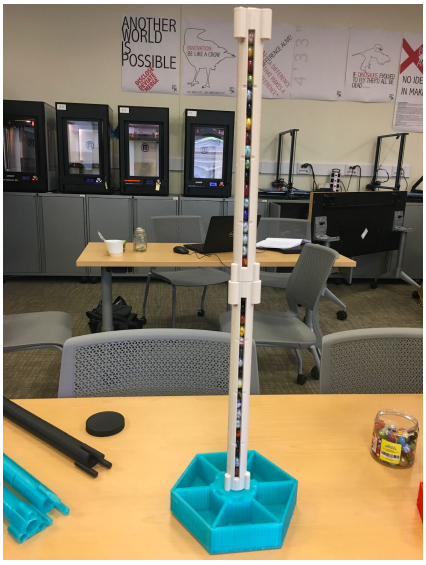
Behind *No Más Caídas* lies a great design story organized around the learning theories of constructivism and constructionism. These theories recognize that knowledge is actively constructed by a learner, with constructionism adding the dimension that the knowledge be constructed during the process of making a shareable object (Harel & Papert, 1991). As we worked towards making *No Más*

Caídas as powerful for sharing as possible, it has developed tremendously in terms of size and colors, and has gone deeper into different functions as we uncovered all of its innate mathematical possibilities. When a child sees the finished product, they could quickly take for granted that it all functions properly as one cohesive manipulative. Because we know the tool personally, we recognize the hard work that has gone into making *No Más Caídas* and are very appreciative of the support of our professor and Josh, our go-tech guy in the lab, as well as the community at the 3D lab who gave us feedback on our tool over casual conversation before and after class.

Initially, we printed a tiny red model as a proof of concept, in order to verify that the proper relationship between the diameter of the tubes, holes, and pegs is a strong one. There was also a transformation behind the designing of the base that supports the towers of *No Más Caídas*. This ensued with a question we hadn't even thought of until the very end of the semester--*how are we going to get the marbles out?* Initially, we designed the square base as a set of "valleys," or tunnels for the marbles to naturally fall into upon release. Dr. Greenstein had a good follow-up idea of making the base a deeper hexagon with the intention of the marbles falling into place more smoothly. Ironically enough, our marbles go in different directions and "fall" everywhere upon release, but when testing the tool out with Maya's daughter, this was a strong feature that provided her with the most excitement.



Pictured above: the many printing iterations of *No Más Caídas*



Pictured above: *No Más Caídas* standing tall today

We also had to decide how high the markings would be based on the size of the marbles. Our professor assisted us with the technology implementation in Tinkercad, after which we all felt like we got very lucky that the markings perfectly line up with the size of five marbles. Another determination

that had to be made was what and how many markings would be added, and the decision of markings by fives was carried over from my (Zoe's) ruler idea, as multiples of five are one of those more appealing sets of numbers to multiply and measure. What we discovered during our interview is that the set of five markings fits perfectly with counting by five's on a child's hands, which serendipitously confirms the mathematical richness of our tool we believed in uncovering during designing.

All of our decisions around the markings were guided by our recognition of the value of the child being the one who provides mathematical meaning to the tool, empowering them to formulate mathematical connections *on their own*. We decided on numberless markings that signify the space of where every five marbles "fall" into place, so that the student could stumble upon the discovery that the markings represent every five marbles themselves. Their action of physically putting the marbles inside *No Más Caídas* are what line the marbles up against the ruler markings, and it is the child who then makes sense of what those ruler markings mean. As discussed in class, it's not that the rulers or other models display the concept to the students, but rather, the students must construct the concept and impose it on the model on their own.

Our last major design decision was a result of the limitations we incurred with the 3D printing technology. Our initial idea was to use clear, see through filament to make it easy for the children using the tool to clearly see how many marbles they were working with. As we kept designing our tool, we kept printing its iterations with the objective to get the measurements right first, and planned to print the tool with the clear filament as the very last step a week before our in-class presentation. Unfortunately, the 3D printer was not able to successfully produce our tool with the original transparent

material as intended, as it did not take to the filament well, like it had with the other type of filament we had been using.

We then faced a huge dilemma: how were the children supposed to see what they were counting if the tool's shell was not transparent? There was a faint shadow of the marble inside the tool, but it wasn't strong enough for the child to see. We presented this issue to our class during our final presentation, and it ended up sparking a productive brainstorm and discussion. We heard some valuable ideas on how we could solve for it, like shining a strong flashlight behind the existing container so the marbles could be seen. We ultimately ended up re-printing our tool with slits down the middle, a provided recommended solution from Professor Fernandez and other students in the class that night, so the marbles could be seen and counted. Our finished product actually hid the first marble, as the slit does not go down far enough to the bottom of where the first marble sits. We did not notice this until we conducted the interview, when Nashalie's counting for one of our initial questions was off by 1 marble; this had us realize that the opening wasn't all the way open down to the bottom, so it covered the initial marble. However, Maya's husband has since fixed this minor flaw, and our tool is operating beautifully as intended.

TASK STATEMENTS

The below tasks were originally written for an elementary school aged child to interact with using No Más Caídas. They were then adjusted for Nashalie, Maya's five-year-old daughter, and presented to her along with the tool.

1. Nashalie went to the Dominican Republic and brought back several ***rocas hermosas*** and wanted to combine them with Zoe's rocks from the mountains in New Jersey. Today we want to find out

how tall both rock collections are together, on top of each other. Zoe stacked her rocks, which were 9 rocks tall. Nashalie had stacked her rock collection, and they were 11 rocks tall. Can we figure out how tall they will be end to end?

   a. We also did an extension including a variation of different numbers (8 and 6) and changed the names to Ayden and Adrian, her brothers.

2. We are going to drop 12 marbles in here; how high up do those marbles go?
3. On Zoe's first trip to the Dominican Republic, she encountered un ***Rhinoceros Iguana muy grande*** that she measured to be one foot tall. How many marbles tall is the ***Rhinoceros Iguana***?

To test our *herramienta*, we wrote story problems to incorporate our learnings from class around the importance of providing students opportunities to derive meaning from context on their own. We talked about how the whole point of giving context is so that they can act on that context, and make sense of it. Once children actively make sense of the problem, they open the window to engage with the mathematics. As one of the readings from class claims, "If we want students to understand mathematics, it is more helpful to think of understanding as something that results from solving problems, rather than something we can teach directly." (Hiebert 1997). Our tasks allow us to aim to position the manipulative as an easy-to-use and engaging device the child can use to respond to the mathematical questions and problems posed in a way that gives them agency over their own learning. As such, we sought to create a more relatable problem-solving context in which it would make natural sense for the child to use *No Más Caídas* to carry out the tasks presented.

We chose the language of our problem deliberately in a way that could represent the vertical nature of our tool to set it apart from other counting tools and manipulatives. One of our classmates, Sam, posed the question of why we couldn't just allow the child to use our base as a counting container/sorter, and our measurement task is proof that the actual tool needs to be used to solve this (i.e., asking questions like, "how high" and "how many marbles tall"), as the child cannot stack marbles on top of each other to measure without it. We tried to bring the context to life during the interview by making additional modifications, and told Nashalie that the marbles represented the rocks for this task. We could also assign Zoe and Maya's marbles two different colors, representing something unique to each of them.

FINDINGS

Maya. Nashalie estuvo super entusiasmada durante la entrevista. Para ella solo era un juego, no hubo espacio para aburrimiento o cansancio. Todo empezó como un simple conteo en el cual pensamos que ella se detendría alrededor de las 20 canicas. No fue así. Ella insistió y contó hasta 36. Nashalie queria mas y mas, por lo que procedimos a plantear los tres problemas antes presentados. Debido a su poca experiencia con números, decidimos presentar cada problema con un lenguaje llano y claro con el cual ella entendiera y se atreviera a continuar. Estos problemas fueron presentados de manera que hicieran sentido para ella. Nashalie estaba trabajando con números más grandes a los que ella estaba acostumbrada, más su alegría y dedicación al usar nuestra herramienta favorecio nuestra entrevista. Nashalie fue capaz de resolver los mismos problemas presentados a Verónica aun con números más grandes.

Zoe. There is something quite inherently powerful in seeing what you developed be put into action. I had met Nashalie many times before, as Maya would bring her to campus so that her husband could pick her up before class. It was nice to have an established relationship with the student who we wanted to learn from our tool, as her happiness as a result of the tool we created is so much more meaningful. Nashalie was already excited to be at her Mom's school, and absolutely loved being in the Maker Lab; the professor who runs the lab even gave her a toy and a few manipulatives to take home. As soon as we brought out *No Más Caídas*, Nashalie squealed with delight, and was jumping up and down as she took each marble, put it into the tool, and heard it slide all the way to the bottom. We gave her about 15 minutes of straight "playtime" with the tool, and she was able to count to 36, which greatly impressed Maya since she thought 20 would be the highest she could count to.

Pictured above: Maya, Nashalie, and Zoe standing tall with *No Más Caídas* in the MIX Maker Lab

When presented with the first problem, Nashalie picked up each marble, one-by-one slowly and with care. She seemed to be truly enjoying the process of putting each marble into the tool, watching it fall to the bottom. She also kept track of her counting with ease, and when asked how many total rocks there were, she responded with "20!" immediately. We did not know if this was because the 20 lined up with the marking exactly, so we presented her with another problem using different numbers (6 and 8). "Varying the numbers in a story problem is one way to differentiate the content challenge of such mathematical tasks" (Bray 2009), which is what we accomplished by changing the numbers for Nashalie. Nashalie was able to answer that there were 14 rocks total, and we used the fact that the marbles fell just one short of the 15 marking to teach her about number decomposition. We asked questions like, "What do you notice about that marble and how far it is away from the marking?" and Nashalie was able to visualize that 14 is one less than 15, assisting her learning around the nested nature of numbers, as we intended.

Since the tasks we presented Nashalie with were going well, Maya and I decided to make up a division problem that I had used previously with Yasmine during our dividing problem solving interview. After asking her what her favorite dessert was (cupcakes), we told Nashalie there were 24 cupcakes, and we wanted to make three plates full of the cupcakes. We then asked her, how many cupcakes were on each plate? We ended up using the empty marble containers as a representation of plates, so Nashalie could count the cupcakes. Nashalie counted 24 marbles using our tool, released them, and then started distributing them evenly across the three "plates." Once there were no more marbles left, Nashalie took the 8 marbles from one of the containers, and dropped them into the tool to answer that there were 8

cupcakes on each plate. Maya was stunned that her preschool child was doing the type of mathematics she once thought was beyond her reach.

We then provided Nashalie with our last problem, around measuring how high one foot is. For this problem, Nashalie needed more assistance in equating what one foot represents in terms of our tool: we told her that one foot equals 12 marbles, and then she put in all 12 marbles to see how they all filled one complete tube of the tool. She enjoyed the act of discovering how many marbles are in one tube very much. In fact, Nashalie was engaged for a total of 50 minutes, which we found significant since typically Maya, who is a preschool teacher, sees that students hardly concentrate after 15 minutes. Often times, 20 minutes is too much, and when we asked why she liked our tool, Nashalie said because this is a lot of fun.

Maya said Nashalie was more excited by the math of our tool than the math she is used to experiencing in the classroom, as what Nashalie is accustomed to mathematically is her teacher writing numbers on the board, or counting using a numberline. Nashalie's typical experience of doing mathematics mirrors how our book describes traditional mathematics, claiming, "Children in traditional mathematics classes often describe mathematics as imitating what the teacher shows them. Instructions to students given by teachers or in textbooks ask students to listen, copy, memorize, drill, and compute. These are lower-level thinking activities and do not adequately prepare students for the real act of doing mathematics" (Van de Walle 2014). *No Más Caídas* is our tribute to what this "real act" of doing mathematics is like, which begins with engagement. When we teach students, engagement is always the number one challenge, and during our interview, we never lost Nashalie, and Maya and I were very excited that we had her attention the entire time.

REFLECTIONS

We didn't want to just re-create another regular math manipulative that we've used in class before, like the katie cubes--we wanted something different and exciting, while also being accessible, inexpensive and easy to assemble and use. Creating and refining *No Más Caídas* was perfect for this purpose, as it is intended to present math in an engaging way. *No Más Caídas* encourages eagerness to learn by making the act of counting, of doing math, glorious and playful. The making of *No Más Caídas* provided us with a powerful learning opportunity as teachers who are studying and striving to be better. Although we always knew teaching is about modifications, this project allowed us to actually experience what that feels like for ourselves as teachers who are making a real, tangible learning tool for real students. Not only did we have to modify design decisions based on technology constraints and how users reacted to our tool, when we conducted the interview with Nashalie we modified the word problems, and also used what was around us - like the marble jars - to help Nashalie visualize the problem better. Our tool wasn't the only solution *to* learning, but rather a gateway *for* learning. We constructed this gateway ourselves as learners in the same spirit we aim for with our students as their teachers. As such, this project provided us with an authentically real educational opportunity to understand what it means to empower the learning of mathematics through the making of physical tools.

Maya. Nashalie disfruto cada problema que se le presentó. Al contrario, ella quería continuar usando nuestra herramienta. En mi opinión, Nashalie no iba a ser capaz de resolver los problemas debido a que era muy pequeña y su poca exposición a los números. Carpenter describe como los niños a temprana edad preescolar despiertan curiosidad que envuelven cantidades (1997). Preescolar les

permite ser expuestos matemática básica, tal como: identificación de números, correspondencia al contar, a parear cantidades con sus respectivos números, suma y resta simple, patrones, comparación entre cantidades y un sin número de escenarios para qara que estos sean capaces de llevar sus conocimientos a la vida diaria. Nashalie demostró que no hay límites para aprender matemáticas, que no importa cuán joven eres, *No Más Caídas* llamó su atención y la mantuvo cautivada durante la entrevista. Debo decir que me sorprendió al ver la reacción de Nashalie cuando le presentamos nuestra herramienta. Gracias a sus experiencias previas, ella pudo manipular las canicas y los tubos para resolver los problemas. Cada experiencia de su vida en preescolar (contar los platos para el almuerzo, estudiantes en la línea, manipulación de objetos) facilitó su desenvolvimiento con la actividad realizada durante la observación. Me gustaria ver como reaccionarian mis estudiantes de prek-3 al usar *No Más Caídas.* Que harian estos con los tubos? Contrarian las canicas? Hasta que numero contarán? Probablemente, su reacción sería un differente a la de Nashalie. Mis estudiantes tienen de 3 a 4 años y solo los más avanzados serán capaces de contar, otros jugarian con la herramienta y probablemente podrian estas en sus bocas.

Zoe. As a graduate student, a secondary mathematics educator-in-training, I have never envisioned myself teaching math to a child so young like Nashalie. Currently, in my TA duties for the undergraduate Algebra course at Montclair, I teach to the long faces that my students give me, telling me without words, "*I don't want to be here. I hate math.*" I have grown to understand that the dismal looks are not solely the product of my teaching, but rather more so a representation of their previous experiences with traditional mathematics, a deep-rooted web of fear and anxiety I am only beginning to untangle. Through this project I've noticed an interesting dichotomy between the mathematical

experiences of my undergraduate students and those of elementary school aged children like Yasmine and Nashalie. Counting is one of the most fundamental processes of mathematics that guides the play of children everywhere, yet as children get older it becomes lost as a mundane task on a calculator. Counting is required in numerous and complex mathematical situations like those my Algebra students face where, unfortunately, the pretentious value placed on efficiency and speed tends to overshadow underlying mathematical beauty. This non-traditional project has come at the right time in my second semester, and has fortunately allowed me to put the hope back in teaching after many moments of self-doubt and questioning of mathematics education. Witnessing Nashalie's ecstatic reaction to *No Más Caídas* was an immensely positive change from the lack of learning and enjoyment I see my Algebra students experience in their emporium-style course. The gatekeeping importance of math in college and post-graduate careers has been my reason for teaching older children, but seeing such a younger learner delight in learning math opened up new possibilities for teaching for me. I loved being a part of Nashalie's playing with our learning tool, feeling proud that I literally had a hand in making that mathematical moment possible for her.

Zoe & Maya. This project experience provided us with many of the lessons that come with teaching math, including remembering how children portray a special type of thinking, one that drives their curiosity and desire to make sense of the world around them. Our navigation of the intricate weave of design choices was directed by our knowledge gained in this course about how learning works: the evolution of our work represents the power behind making resources to help encourage and strengthen that building of our own understanding from within. Such an evolution stands for progress, for growth: regardless of what we may originally envisioned or intended to print as a new, physical way for teaching

math, the process of constructing our tool challenged not only our initial vision, but also continues to challenge the traditional way of teaching math. We learned more about how to embrace the unexpected, a core component of teaching *for* understanding rather than teaching *to* know. As designers of instruction, we did what the best math teachers do every day in their classrooms: take an idea for learning, utilize whatever resources we have around us, and make it come completely alive through the teaching and learning moments with our students.

"Piaget said that to understand is to invent. He was thinking of children. But the principle applies to all of us." -Seymour Papert

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