Project Rationale

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**all names and identifiers have been masked/changed to retain anonymity*

Describe your manipulative to date. What was especially important to you as you designed your manipulative? Were there features that you had to let go of? Please describe these.

After our first interview, I realized that Salih is having trouble with decimals. I wanted my manipulative to be interesting to him. So, I decided to design a decimal snake, just like a number line in a 3D form. The decimal snake will be consisted of 10 pieces. In each piece there will be 10 small increments representing a unit. Having 100 units on the snake will help us to represent decimals in tenths and hundredths place. I wanted to split up each small increment into 10 more smaller increments so we could tackle questions with thousandths digit but, because of the limitations of the printer I decided to keep it up to hundredths.

What mathematical task(s) are you thinking of presenting to your child with your tool? (FYI: This item will be graded in accordance with the genuine reasoning the task presents your child. Tasks that are focused on memorizing or rote algorithmic applications will not receive credit. Please come and see me so I can help you with this if you are struggling).

First, I will be giving him a set of decimal numbers and ask him to order them. Second, I will have open ended decimal operations problems that require place value understanding.

TASK 1

Name and compare.

a. 0.48 and 0.6

TASK 2

1. Write each of this set of numbers in the correct box. The box on the left is for numbers smaller than 5.5. The box on the right is for numbers bigger than 5.5.

5.7 5.35 5.02 5.9 5.24 5.47 The first one has been done for you.

5.24	

2. Which number is nearest to 5.5? _____

3. Explain how you figured this out. (using our decimal snake)

_____

4. Write down a number of your own that is bigger than 5.24 and smaller than 5.35.

_____

5. Write the numbers in order from smallest to largest.

_____

6. Explain how you decided which was the smallest number.

_____

TASK 3

Jacob and Salih each timed their own quarter-mile run with a stopwatch. Jacob says that he ran the quarter mile in 2.47 minutes. Salih was more accurate in his timing, reporting that he ran the quarter mile in 2.8 minutes. Who ran it the fastest and how much faster was he?

How could a child reason about this task *without* your manipulative? Describe possible approaches or solutions and hiccups.

TASK 1

Possible Solutions

One of the ways to reason about this task is using 10x10 blocks. You can easily point at the area of 0.48 as 4 tenths and 8 hundredths. Likewise, you can address the area of 0.6 by showing 6 tenths on the 100 block. Since 6 tenths takes up more space then, 4 tenths and 8 hundredths you can say 0.6 is greater than 0.48.

One can also use 10x10 grid and color in each number in two separate grids. If you assign different colors for each place value, you can easily see the difference. For example, let tenths be

yellow and hundredths be green. There will be 1 more yellow and 2 more green in 0.6 than 0.48 which represents the 0.12 difference.

Another way to reason about this task is using the accurate decimal language. 0.6 is 6 tenths and 0.48 is 48 hundredths. Since 6 tenths is greater than 48 hundredths one can come up with the solution of 0.6 is greater than 0.6.

Hiccups

One hiccup might be “Longer is larger”. This is a common error for the students who couldn’t establish the place value understanding of decimals and they try to apply their whole number place value knowledge to decimals. Under this misconception, they would say that 0.48 is greater than 0.6. (Desmet, Gregoire, & Mussolin, 2010; Steinle & Stacey, 2004a, 2004b)

TASK 2

1. One can use 10x10 blocks or grid two represent each number. In all six numbers the whole is 5, the same. We are basically comparing the decimal portion of each number: 5.7 5.35 5.02 5.9 5.24 5.47 to 5.5. Seeing the pictorial representation of 5.5 will help students reason about the rest of the numbers. They can compare **5.7** with 5.5 and see that 5.7 has 2 more columns(tenths) colored in than 5.5 which makes it **greater than 5.5**. Similarly, **5.35** will have 15 less boxes colored than 5.5 which makes it **less than 5.5**. When you compare 5.5 with 5.02 you can see **5.02** has 48 less colored boxes(hundredths) than 5.5 which also makes it **less than 5.5**. Next **5.9** will have 4 more columns(tenths) colored than 5.5 which makes it **greater than 5.5**. The next number **5.24** will have 26 less boxes(hundredths) colored which makes it **less than 5.5**. and finally, **5.47** will have only 3 less boxes(hundredths) than 5.5 which also makes it **less than 5.5.**

In the left hand box: 5.35 5.02 5.24 5.47

In the right hand box: 5.7, 5.9

2. 5.47 is only 3 little boxes(hundredths) away from 5.5. It rounds to 5.5 and no other number does.
3. Using the 10x10 grid you can visually prove that 5.47 is the closest number to 5.5.
4. Answers may vary. Any number larger than 5.24 and less than 5.35. For example: 5.25, 5.26, 5.27, …., 5.30, 5.31, 5.32, …until 5.35.
5. From the 10x10 grid representations you can see that the order from least to greatest must be as follows: 5.02 5.24 5.35 5.47 5.7 5.9
6. You can compare the colored regions of each number and see that 5.02 has the least number of colored boxes (only 2 hundredths) among all the numbers. You can also look at the first number after the decimal point and choose 0 because it is the smallest.

Hiccups

The hiccup for this task might be "Longer is larger". "This is the most common initial error—students select the number with more digits as largest. This is an incorrect application of whole-number ideas, as students just look at the number beyond the decimal point and judge it as they would a whole number". (Van et al., 2013, p.347) Under this misconception, they could say that 5.47 is greater than 5.7.

TASK 3

In this problem we are comparing 2.47 and 2.8. The reasoning piece of the problem is that the kid with the least amount of time will be the fastest kid. When you compare 2.47 with 2.8 you automatically see that tenths digits are different. Since 4 tenths is less than eight tenths, we say

2.47 is less than 2.8. We can represent each decimal on a 10x10 grid. We will be coloring 47 hundredths for 0.47 whereas we will color 80 hundredths for 0.8. So, it will be clear that 2.47 is less than 2.8.

Hiccups

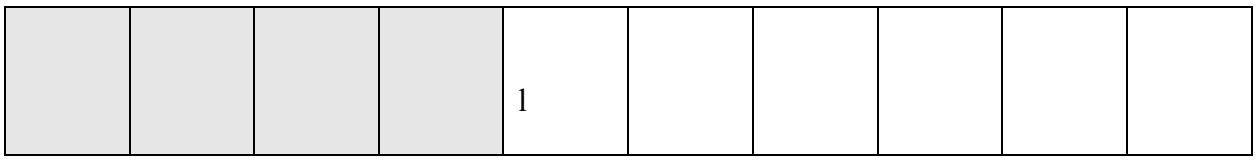
There might be a reasoning hiccup for this word problem. It might be confusing to relate the fastest kid should have the least amount. Also, "the longer is larger" hiccup is possible. One can say 2.47 is bigger because it is longer than 2.8. (Desmet, Gregoire, & Mussolin, 2010; Steinle & Stacey, 2004a, 2004b)

How could a child reason about this task *with* your manipulative? Describe possible approaches or solutions and hiccups.

The decimal snake is divided in 100 sections. A child using my decimal snake can reason similarly with using a 10x10 grid or 100 block. The only difference is every ten group are placed horizontally on the decimal snake. Whereas on 10x10 grid every ten block is placed vertically.

TASK 1

On the decimal snake each increment represents one hundredth. Every ten increments are grouped in one piece of snake which represents the one tenth of the snake. 0.6 is 6 groups of tenths pieces together. 0.48 is only 4 groups of tenths and 8 hundredths from the 5^{th} piece of the snake. This helps students to compare 6 groups is greater than 4 groups and some more (not equal to 5 groups).



0. 48

0.6

TASK 2

Since all the numbers have 5 in their ones digit, we can compare the decimal portion of each number.

0.02

0.24

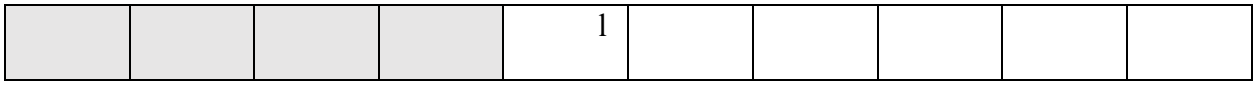
0.35

0.47

0.7

0.9

TASK 3



0.47

0.8

Using the snake you can draw these two diagrams and you can clearly see that 0.47 is “shorter” than 0.8. So the person with the shortest amount of time will be the fastest. The decimal snake will help the students to visually see that the time is “shorter”.

Can you think of other possibilities for reasoning that your manipulative affords a child? (Actually, maybe you could all ask your child this question at the end of your final interview! That is, you could all ask, “Can make up another math problem you could solve with this manipulative?”).

The decimal snake also can be used as a number line. Any operations that can be explained on a number line can be easily performed on the decimal snake. For example, integer operations can be performed other than comparing numbers and decimals.

How does the design of your tool reflect your understanding of what mathematics is and how learning happens? (For this question, I would like you to read about your topic in the Van De Walle text and incorporate at least one reference on your topic into this section. I will be looking for a reference in the write up of this paper.)

During my first interview my child stated that he was struggling with decimals. So, I started thinking about a tool that can help him build a conceptual understanding of how decimal numbers are constructed. (Van De Walle, 2013, p.347) I needed a shape with 100 pieces so I can give him examples on tenths and hundredths. During one of my informal interviews I realized that he likes snakes. So, I decided to create a tool that will attract his attention and have a mathematical meaning to it. I created the first task having the five indicators of decimal understanding (Cramer et al., 2015) in mind. The first indicator was using the precise mathematical language when working with decimals. I simply asked him to read and compare *0.48 and 0.6*

My second task has 6 sections in it. There are six sections which will help me to understand his number sense and reasoning with decimals. My third task is an open-ended math problem which requires place value understanding of decimals and may be operations with decimals. I ordered each task in an increasing difficulty level to see in which step he needs support.

What would it mean for your project to be successful in terms of learning? What would it mean for your project to not be unsuccessful in terms of learning?

I created the decimal snake in the hopes that it helps him visualize the meaning of each decimal. So, if he can relate each decimal and represent them on my decimal snake that means my project is successful. However, if he cannot connect his decimal understanding to the decimal snake than my project is not successful.

References

Desmet, L. Gregoire, J., & Mussolin, C. (2010). Developmental changes in the comparison of decimal fractions. Learning and Instruction, 20, 521–532.

Kathleen Cramer, Debra Monson, Sue Ahrendt, Karen Colum, Bethann Wiley, & Terry Wyberg. (2015). 5 Indicators of Decimal Understandings. *Teaching Children Mathematics, 22*(3), 186-195. doi:10.5951/teacchilmath.22.3.0186

Van de Walle, John A. (2013). *Elementary and middle school mathematics : teaching developmentally.* Boston :Pearson