ReMaking Teacher Learning: Designing Objects-to-Teach-With to Promote Mathematics Education Reform

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ABSTRACT
In this proposal, we describe an ongoing research project that incorporates a Making experience into a graduate-level specialized mathematics content course offered to prospective elementary mathematics teachers (PMTs). Through project data and personal narratives of two teacher educators (TEs) and two PMTs who are participant-researchers on the project, we present a case for the transformative power that design and Making have within the space of mathematics learning and teaching. Within the reflective and participatory creative processes associated with the Maker experience and the inquiry-oriented learning environment the TEs cultivated in the course, we found that the class provided the PMTs with an opportunity to *remake* their personal relationships with mathematics and mathematics teaching as rooted in exploration and creativity, and imbued with dynamic and deep understandings of mathematical ideas.

Keywords
Making; Mathematics Education; Teacher Preparation; Elementary Teaching

2.1 Description of your setting
Our work took place across two sections of a specialized graduate-level mathematics content course for prospective elementary teachers taught by two different instructors in a large, suburban, state university in the northeast United States over a period of two semesters (Spring and Fall of 2019). The first class consisted of thirteen students and the second class consisted of fourteen students. Although the university is a Hispanic Serving Institution (HSI), the majority of the students in each class were white. Over 90% were female. These demographics are typical of the prospective elementary teacher population. In both classes, the students brought with them a variety of experiences in teaching; some had already been teaching in the classroom for years and some had no teaching experience in a classroom at all. The students in the class were prospective mathematics teachers (PMTs) of grades K - 6, with some pursuing graduate-level coursework toward their preparation or development in the teaching of elementary mathematics, some working toward their certification in middle grades mathematics teaching, and some studying to gain a dual certification in K-6 and teaching students with disabilities. In our experience, many of the students entered this course with a held belief that mathematical activity and learning is mostly restricted to rote procedures and single solutions. In addition, many of them believed that mathematics is isolated from other disciplines, and has little relevance outside of school. By providing the PMTs with a Making experience in the context of the course, we hoped to confront this narrow view of mathematics pedagogy and support the development of a more inquiry-oriented one. The Making experience also provided a challenge to the students in the class in that many of them had little to no experience in design, Making, or 3D printing or their related tools and processes.

2.2 Description of the educational experience
We set out to study how incorporating a Making experience in a mathematics education class would inform PMTs' often held model of mathematics teaching typically governed by rules and procedures as they enter into teacher preparation programs. In order to do so, we aimed to 1) describe the impact of PMTs' participation in this design experience on their knowledge and identities, and 2) design curriculum and materials for use in teacher education settings that support the PMTs' Making experience. Through PMTs' participation in this design experience, we aimed to guide their conceptual, curricular, and pedagogical thinking and practices toward types that support learning and teaching mathematics with understanding. We hypothesized that the Maker experience would afford unique pathways of diversified engagement opportunities for PMTs to understand the complex challenges of teaching and learning that align with the kind of reform-based, progressive, and inquiry-oriented pedagogy that we aimed to cultivate in our future teachers. In other words, we proposed that this intersection of digital fabrication technologies, human-

centered design practices, and constructionist orientations to domain-related thinking and learning would provide an open-ended and collaborative space that allows teacher educators (TEs) to teach in a way that (re)humanizes mathematics and mathematics teachers through creative and participatory practices.

Situated in an instructional context in which the TEs practiced an inquiry-oriented pedagogy based in a constructivist theory of learning, the course engaged students in a Making experience defined by the following task: "The purpose of this project is for you to 3D design and print a new physical tool (or 'manipulative') that can be used in teaching a mathematical idea, along with corresponding tasks to be completed by your student(s)." All assignments and tasks were designed with an iterative nature in mind, thereby engaging the PMTs in the questioning, risk-taking, false starts, and revising that are inherent to problem solving within a constructivist framework. The PMTs learned to use Tinkercad (http://tinkercad.com) to design, refine, and print their manipulatives with the intention to help elementary-aged children learn mathematical concepts. The design of this tool – and corresponding tasks meant to mediate users' engagement with it – aimed to reflect a) PMTs' knowledge of what it means to do mathematics and how we learn with physical tools, b) their knowledge of elementary-level mathematics content, and c) their perspective on pedagogy and curriculum in mathematics education. In addition to the design of the tool, the project had three written components: 1) an "Idea Assignment" that described initial thoughts about the manipulative they desired to create, 2) a "Project Rationale," which was an account of the PMTs' understanding of how their design might reflect the learning of the mathematical idea(s), and 3) a "Final Paper/Reflection" that described the manipulative's purpose, corresponding tasks, and findings from interviews conducted by the PMTs with their focus students.

3. CONCLUSION

3.1 Results

We present the results of this educational Maker experience from the perspectives of both TEs and two PMTs, all four of whom are participants in this research and contributing authors of this proposal.

Teacher Educator 1 (TE1): The idea for the Maker project emerged from the first teacher educator's (TE1) experience teaching a doctoral-level design course in mathematics education in which the possibilities for digital design and fabrication were explored. As a result, TE1 entered into this experiment with confidence in his abilities to support PMTs' design practices and with genuine optimism about the prospect that the Making experience would be promising for supporting the development of PMT's constructivist-oriented pedagogy. Having said that, what was particularly striking for TE1 about the outcomes of the experience was the diversity of routes the PMTs took to the design of the physical, mathematical, and pedagogical components of their projects. For example, one group – whose story is told from the perspective of PMT2 later in this proposal – leveraged the native linguistic, cultural, and historical knowledge of one its participants to design a tool for playful counting and organizing that contributed to a particularly humanizing design experience for her. Another designed "fraction circles" that enabled her to finally achieve an understanding of fraction division – a topic she had initially hoped to design for and then later discarded when she was discouraged by it – even though she had not designed the tool for that express purpose. And another, PMT1, designed a "fraction orange" that she used in an impromptu interaction with her father as they engaged in sustained, collaborative, mathematical, problem-posing activity that demonstrated the dynamic and contingent nature of their mathematical knowledge. The most formative aspects of these outcomes could not have been predicted, nor could we have anticipated the significance of this question, which we're all now wondering about: *What was it about the culture of the Maker environment – and the knowledge and experiences of PMTs who worked within it – that interacted with and informed their designs?*

For TE1, two things stand out as promising about the Making experience. First, it seems to be the case that when prospective teachers enact agency over the design of their own manipulatives and the corresponding curricular materials associated with them, they are able to demonstrate their capacities as knowledgeable and effective designers of mathematical instruction within learning environments that they themselves have generated. Second, the authentic, open-ended nature of the design task situated in a space of technological possibilities not only enables PMTs to assume such agency, it also serves to reveal a wealth of data from which we can learn about what they're learning and who they're becoming in relation to mathematics and mathematics teaching.

Teacher Educator 2 (TE2): The second teacher educator (TE2) was initially anxious about incorporating a Making experience into her class. She hadn't taught the class in over fifteen years and was worried about learning a new design software (Tinkercad) at the same time she was drawing up new class lessons. She also carried with her a long-standing philosophy about learning mathematics, namely, that eliciting and using students' perspectives to explore problems and pose investigations can empower them and help them appreciate the value of *their* thinking in the problem-solving process. For her, students' perspectives could include errors, struggles, or unanticipated problem-solving paths, and she wondered how this philosophy would interact with the design experience.

For TE2, the design experience became a new setting for instantiating her philosophy. That is, the setting provided a novel forum in which to investigate the tangents and struggles that can arise during problem solving as she participated with her PMTs to design their manipulatives. She experienced the kind of partnering between teacher and student (i.e., between TE and PMT) that involves all parties in genuinely exploring the issues and struggles that arise during the course of solving problems, and she was delighted at the insights and learning she and her PMTs shared. For example, when one PMT opted to "design" a manipulative that already existed, TE2 trusted that the experience would still provide for meaningful learning (and it did!). When another PMT designed

multiple solid shapes to see which "peg" would fit neatly into a hole, TE2 learned how to incorporate all the solids (even the ones that didn't fit) into the design experience! And then there was the time when she watched as a PMT's design of a dragon that loses its heads and spawns two heads at a time in its place spawned the notion of exponential growth for that PMT who had not encountered exponential growth before! As TE2 watched the recorded interview videos as these and all her PMTs used their manipulatives to navigate the special circumstances of learning that arise with children, she observed how each teacher made themselves vulnerable to at least one of the special, mathematical insights their child introduced, and navigated that insight to important learning for both themselves and the child.

Prospective Mathematics Teacher 1 (PMT1): The Making experience was a transformative one for PMT1, not only in her orientation to mathematics and mathematics teaching, but also in relation to her own identity as an educator and student. PMT1 entered the Spring 2019 class with a background in the arts as a graduate student working toward her certification in elementary education and as a teacher of students with disabilities. She had little experience in mathematics as a college student, and no prior experience in mathematics teaching. As the class progressed, the design process, in conjunction with inquiry-oriented coursework of TE1's curriculum, helped her to articulate her complicated (albeit exciting and dynamic) relationship with mathematics, as well as her developing pedagogical philosophies. She set out to design and 3D print a manipulative to help a fourth-grade special education student compare, add, and subtract fractions. Her background in the arts made the design process an intuitive one for her, and she created a "fraction orange" as a tool for manipulating, composing, and decomposing nested "slices" to explore and better understand halves, quarters, eighths, sixteenths, thirds, sixths, ninths, and eighteenths, and the relationship between a whole and its parts. While the orange proved to be useful in that sense to the student she was working with, PMT1 found that the tool completely uprooted her notions and understanding of fractions when she used it with other adults to solve fraction division problems. It was through these interactions with the tool she designed that she, and the other adults, were moved (in a profound way) to reexamine their relationship to fractions and mathematics.

In addition, the opportunity to engage in the creative process in the context of a math education course was particularly powerful for PMT1, as it illuminated the interdisciplinary connections between the arts and mathematics that she had never explored previously in her prior education. Additionally, these insights into her own mathematical understandings provided her a new perspective on what a rich learning experience of mathematical concepts can entail – one that resists a traditional, procedural mathematics teaching and learning experience and embraces a creative, inquiry-oriented, and collaborative one. All of these new and exciting insights into her relationships with mathematics, teaching, and learning that resulted from creating and using her tool would ultimately lead her to join the research team in order to further explore how incorporating creative processes into the context of mathematical learning experiences might impact curriculum, teachers, and students in important ways.

Prospective Mathematics Teacher 2 (PMT2): It had been four months into PMT2's graduate studies on mathematics education when she enrolled into the Spring 2019 class. During these initial four months, she was strictly looking at the project from a researcher's perspective, focusing her analysis on how the Making experience could elucidate all the ways of knowing a PMT might bring to the learning of teaching mathematics. But enrolling as a student in the class meant that she was also a participant-researcher in the study, and she had the opportunity firsthand to design and 3D print her own manipulative for mathematical learning—a tool she and her co-Maker-turned-friend named *No Más Caídas*. PMT2 and her partner designed *No Más Caídas* with the ideals of simplicity and organization at its core, aiming to create a tool for children to play and count with marbles. In addition to giving their tool a Spanish name, selecting marbles as the counting objects became a mark of PMT2's co-Maker's Dominican culture.

The non-linear design process behind *No Más Caídas* included 10+ printing iterations. It also represented their (sometimes uncomfortable) embrace of the unexpected and their navigation of the intricate weave of design choices, which were guided by the constructivist learning philosophies of the class. The professor, TE1, pushed PMT2 and her co-Maker to create a tool that was more than just a clear representation of a collection of objects. As they continued to question and discuss their ideas with each other, the partners kept their goal of challenging and resisting the traditional ways of teaching math at the forefront of their design decisions, positioning and advocating themselves as designers of mathematical instruction that teach for understanding rather than teach to simply "know" memorized shortcuts and algorithms. They decided on numberless markings that signify the space where every five marbles "fall" into place on the tool so that the child could stumble upon the discovery that the markings represent every five marbles on their own. This physical design of their tool – paired with the intentional design of open-ended tasks – provided carefully constructed support to the child from their clinical interview to not just count, but to also engage in meaningful number decomposition.

While tangents became a common occurrence for PMT2 and her co-Maker within their design process, all the surprises and false starts that ensued ultimately taught them an appreciation for utilizing the environment and community around them to Make, learn, and teach – both separately and all at once. They wrote of these takeaways in their final paper: "Our tool wasn't the only solution *to* learning, but rather a gateway *for* learning. We constructed this gateway ourselves as learners in the same spirit we aim for with our students as their teachers. As such, this project provided us with an authentically real educational opportunity to understand what it means to empower the learning of mathematics through the making of physical tools."

Beyond the experiences of these two students, we discuss what we learned from case study analyses of the broader impact of the Maker experience on the local community of the other preservice teachers who participated. Those PMTs who described a shift in their orientation toward mathematics and teaching mathematics shared that while they originally entered the class with a negative relationship to mathematics, the Maker experience and coordinated work with students who used the tools they made ultimately

left them with deeper understandings of the mathematics and knowledge of how they might teach it to their future students. Evidence in their written work corroborates these assertions and also speaks to the kind of "fun," exploratory, and mathematically meaningful learning environments they hope to generate – one that embraces the resilient spirit of mathematics that can often be clouded by traditional mathematics teaching.

Finally, the TEs learned from the outcomes of the reflective and participatory creative processes associated with the Maker experience, as well as the nature of the learning environment in which it was implemented, that it served as a medium not only for the cultivation of the kind of pedagogy that the course had always aimed to promote, but also for the rehumanization of PMTs' relationships to mathematics and mathematics teaching. As a result of these impacts, the curriculum has been revised to include a Maker experience in future iterations of the course. Furthermore, the curriculum is being prepared and packaged for broad distribution to the mathematics teacher educator community for use in other courses.

These results can be further explored through our website, www.teachermakers.com, where we present the PMTs' manipulatives as well as other resources related to the Making experience.

3.2 Broader Value

The intellectual merit of this project stems from a constructionist orientation to teaching and learning and the proposal that new access to human-centered design practices and digital fabrication technologies will lead to powerful innovation in mathematics teaching and learning. By leveraging the consequent possibilities, this project can generate new knowledge about the contributions that Making can offer for preparing mathematics teachers; for developing their knowledge, pedagogy, and technological skills; and for authoring themselves as effective and agentive designers of mathematical instruction. More specifically, we aim to showcase specific case studies from the point of view of both PMTs and TEs involved in the research to exemplify aspects of the Maker design process that compelled these teachers to grapple with an authentically open-ended situation and how this influenced the ways they understood learning and engaged with learners.

As technologies like 3D printing become more pervasive in schools, teachers whose pedagogies have been informed by Making experiences will be well positioned to develop these experiences for their students and cultivate their peers' and students' 21st century STEM interests and capacities. Further iterations of the Making experience will yield a viable curriculum module that will be made available for widespread use in teacher preparation, and especially in mathematics teacher preparation. This module will engage prospective teachers in an iterative design experience that aims to advance their knowledge for teaching through the digital design, fabrication, and evaluation of new physical tools for use in instructional situations. Additionally, a creative and collaborative design environment for the PMTs' Making is intended to demonstrate the influence of this environment on PMTs' uses of knowledge during teacher education, to cultivate PMTs' identities as designers of mathematical instruction and as agents of curricular and pedagogical reform, and to alleviate the historic trend of anxiety that some PMTs have suffered in relation to their understanding of mathematics and their future implementation of a student-centered model of mathematics pedagogy.

3.3 Relevance to Theme

An underlying intention of creating this Making experience within a preservice teacher education course was to embolden a reform-oriented approach to teaching mathematics within future educators and to resist the traditional procedural mathematics teaching often experienced by and taught by teachers. We *resist* by reincorporating creativity, individual relevance, and opportunity for explorative understanding of mathematical ideas through our Maker experience so that PMTs can begin to (re)form their relationships with mathematics and teaching. In addition, the Maker experience within this context focuses on the value that can be gleaned from the discomfort that is often present within mathematical activity and creative design. Learning to harness and leverage this discomfort through iterative design experiences helps PMTs build *resilience* as teachers and doers-of-mathematics. Moreover, our TEs also learn to build *resilience* in sitting with the discomfort of pushing students according to a pedagogy that requires that they receive agentive intellectual freedom and highly intentional but minimal guidance.

4. BIOS

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